The Demand for Youth: Explaining Age Differences in the Volatility of Hours

Nir Jaimovich, Seth Pruitt and Henry E. Siu*

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Abstract

Over the business cycle young workers experience much greater volatility of hours worked than prime-aged workers. This can arise from age differences in labor supply or labor demand characteristics. To distinguish between these, we document that, for young workers, both the cyclical volatilities of hours and wages are greater than those of the prime-aged. We argue that a general class of models featuring only age-specific labor supply differences cannot reconcile these facts. We then show that a simple model featuring labor demand differences can.

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*Jaimovich: Duke University, Department of Economics, 213 Social Sciences Building, Durham, NC 27708-0097 (e-mail: njaimo@gmail.com) and NBER; Pruitt: Federal Reserve Board, Division of International Finance, 20th St and Constitution Avenue NW, Washington, DC 20551 (e-mail: seth.j.pruitt@frb.gov); Siu: University of British Columbia, Department of Economics, #997-1873 East Mall, Vancouver, BC, Canada V6T 1Z1 (e-mail: hankman@mail.ubc.ca) and NBER. We thank the co-editor, Dirk Krueger, the anonymous referees, Manuel Amador, Paul Beaudry, Larry Christiano, Marty Eichenbaum, Valerie Ramey, Sergio Rebelo, Victor Rios-Rull, William Gui Woolston, and numerous workshop participants for helpful comments, and Josie Smith for superb research assistance. Siu thanks the Social Sciences and Humanities Research Council of Canada for financial support. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. A previous version of this paper circulated under the title "The Demand for Youth: Implications for the Hours Volatility Puzzle."

I Introduction

Labor market fluctuations over the business cycle differ greatly for individuals of different ages. Perhaps the most salient difference is between the volatility of hours worked of young workers relative to the prime-aged (see Kim B. Clark and Lawrence H. Summers, 1981; and Paul Gomme, Richard Rogerson, Peter Rupert and Randall Wright, 2004). While this fact is well known, the literature lacks a quantitatively successful explanation.[1]

In our view, developing such an explanation is important for our understanding of business cycle and labor market dynamics. As an example, the results of Nir Jaimovich and Henry E. Siu (2009) show that workforce age composition has a strong causal impact on employment and output volatility. A theoretical explanation of this lies in understanding age differences in cyclical sensitivity. In addition, understanding age differences in the volatility of hours – and specifically, why young hours are so volatile – leads to an understanding of the volatility of aggregate hours as an important corollary. Accounting for the relative volatility of aggregate hours to output remains as one of the puzzles in real business cycle (RBC) analysis (see Robert G. King and Sergio T. Rebelo, 2000; Gomme et al., 2004).

In this paper, we study this phenomenon through the lens of a neoclassical model in which households and firms optimize, taking prices as given, and interact in competitive spot markets. We view this as an important exercise given the prominence of this framework in quantitative business cycle analysis. Within this framework, age differences in the volatility of hours can arise from factors related to preferences (or succinctly, differences in labor supply), technology (labor demand), or both. We argue that the *joint* behavior of age-specific hours and wages over the cycle provides the necessary evidence to distinguish between these two channels. Specifically, we document in Section II that *both* the volatilities of hours and wages for young workers are greater than those of the prime-aged over the cycle.[2] We view these as our two key labor market facts.

We show in Section III that a general class of models featuring only age-specific labor supply differences cannot reconcile these facts. Generating a higher volatility of hours of the young implies a lower volatility of wages, and vice-versa.

[1] Jose-Victor Rios-Rull (1996), Gomme et al. (2004), and Gary D. Hansen and Selahattin Imrohoroglu (2009) study models with differences in hours volatility owing to life-cycle considerations. They find that such factors cannot quantitatively account for the greater volatility of young hours relative to others.

[2] This approach takes seriously the assumption of spot labor markets in the neoclassical model. One could imagine a model where contracts insuring hours and/or wage fluctuations are provided differentially by age might also rationalize these phenomenon. Nevertheless, we note that existing empirical evidence does not support the hypothesis of age differences in the extent of contracting; see James T. McDonald and Christopher Worswick (1999).

Consequently, in Section IV we present a model that rationalizes the labor market facts by allowing for cyclical differences in age-specific labor demand. Our approach represents a minimal deviation from the standard RBC model, extended to three factor inputs: capital, "young" labor, and "old" labor. We study a model where the elasticity of substitution between capital and labor can differ between young and old. The model features *capital-experience complementarity* in production, when age is equated with labor market experience.[3] We note that our model represents one particular micro-foundation for the differences in the cyclicality of labor demand. A simple alternative is to allow for shocks to young labor input in the production function that are more volatile than shocks to old labor. However, we view this approach as unappealing since it essentially assumes the desired result and lacks the discipline that our approach entails. Specifically, in Section V, we use our model's factor demand equations and estimate the key elasticity of substitution parameters in a manner that does not target the differences in cyclical volatility of age-specific hours and wages.

We find that our capital-experience complementarity model generates relative volatilities of hours and wages across age groups that are similar to those observed in the data. As a by-product, the model also generates a relative volatility of aggregate hours to output that is essentially unity. These results are presented in Section VI. Section VII concludes.

II Age-specific hours and wages

In this section, we document the empirical findings that motivate our work. We first present evidence on the large differences by age in the volatility of hours over the cycle. We then provide evidence on the cyclicality of age specific real wages. These facts allow us to distinguish between models in the analysis that follows.

A Hours

The evidence on the cyclicality of age-specific hours has been extensively addressed in Gomme et al. (2004) and Jaimovich and Siu (2009). We provide a brief summary here and refer the reader to the cited papers for greater detail.

Using Census data from the March supplement of the Current Population Survey (CPS) over

[3]Previous work has emphasized the existence of complementarities between capital and skilled labor, when skill is proxied by educational attainment (see, for instance, Per Krusell, Lee E. Ohanian, Jose-Victor Rios-Rull and Giovanni L. Violante, 2000 and Rui Castro and Daniele Coen-Pirani, 2008). However, Mincerian wage regressions emphasize two important observable dimensions of skills: education and experience. We show that labor market experience exhibits important complementarities to capital at the business cycle frequency.

Table 1: Volatility of Hours Worked by Age Group

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	5.66	2.30	1.92	1.44	1.23	1.49	2.05	**2.33**	**1.23**
R^2	0.80	0.79	0.68	0.94	0.70	0.75	0.13	**0.91**	**0.95**
cyclical volatility	5.08	2.04	1.58	1.40	1.03	1.29	0.73	**2.22**	**1.20**
hours share	3.74	10.85	13.12	26.00	24.16	17.58	4.54	**27.71**	**72.29**
volatility share	12.73	14.83	13.93	24.38	16.67	15.24	2.21	**41.49**	**58.51**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. Hours share is the sample average share of aggregate hours worked by the age group, reported in percentage. Volatility share is the age group's share of aggregate hours volatility, defined as the the average of age-specific cyclical volatilities weighted by hours shares, reported in percentage.

1964-2010 we construct annual series for per capita hours worked for specific age groups, as well as an aggregate series for all individuals 15 years and older. We extract the high frequency component of each series using the Hodrick-Prescott (HP) filter on logged data.[4]

Table 1 presents results on the time series volatility of hours worked by age. The first row presents the percent standard deviation of the detrended age-specific series. We see a decreasing relationship between the volatility of hours worked and age, with an upturn close to retirement age.

We are not interested in the high frequency fluctuations in these time series per se, but rather those that are correlated with the business cycle. For each age-specific series, we identify the business cycle component as the projection on a constant, current detrended output, and on current and lagged detrended aggregate hours; we refer to these as the *cyclical* hours worked series. The second row of Table 1 reports the R^2 from these regressions. This is high for most age groups, even for those whose hours comprise a small fraction of total hours. This implies that the preponderance of high frequency fluctuations are attributable to the business cycle.[5]

The third row presents the percent standard deviation of the cyclical series. The data indicates a pattern of decreasing volatility with age. The young experience much greater cyclical volatility

[4]Since we are interested in fluctuations at business cycle frequencies (those higher than 8 years), we use a smoothing parameter of 6.25 for annual data; Morten O. Ravn and Harald Uhlig (2002) find this to be the optimal value through analysis of the transfer function of the HP filter. One may alternatively use a smoothing parameter of 10 or the bandpass filter as suggested by Marianne Baxter and King (1999) to remove fluctuations less frequent than 8 years. The quantitative results are essentially identical in all cases.

[5]The exception is the 60-64 age group, where a larger fraction of fluctuations are due to non-cyclical movements.

in hours than all others. The standard deviation of cyclical hours fluctuations for 15-19 and 20-24 year old workers is five and two times that of 40-49 year olds, respectively.[6] As a group, the hours of young workers aged 15 to 29 years old are about 1.85 times as volatile as for prime-aged workers, aged 30 to 64 years old.

The fourth row indicates the average share of aggregate hours worked by each age group. The fifth row indicates the share of "aggregate hours volatility" attributable to each age group. Here, aggregate hours volatility is represented by the weighted average of age-specific cyclical volatilities, with weights reflecting an age group's share of aggregate hours. Fluctuations in aggregate hours are disproportionately accounted for by young workers, whose share of volatility is markedly greater than their share of hours. Although those aged 15-29 make up only about one quarter of aggregate hours worked, they account for more than two fifths of hours' volatility. By contrast, prime-aged workers aged 30 to 64 years account for about three quarters of hours, but a little less than three fifths of the volatility.

A.1 Participation

Young individuals might display greater cyclicality of hours worked relative to the prime-aged because of labor supply considerations; for instance, they may face different trade-offs between market work and home production, or possess a greater degree of insurance through parental ties. These possibilities indicate that if labor supply differences are of primary importance, the cyclicality of labor force participation should be more pronounced for the young. To explore this, we note that changes in per capita hours worked can be viewed as being due to changes in either hours per labor force participant, or the number of the labor force participants per capita. We refer to the former as the *hours margin*, and to the latter as the *participation margin*.[7] If the participation margin is the main driver of hours variation for the young, then one could argue the practical necessity of explicitly modeling labor supply differences, and specifically, age differences in the participation decision. If not, it would indicate that to a first-order, the primary factor generating age group differences is to be found elsewhere.

[6] These results corroborate the findings of Gomme et al. (2004) and Jaimovich and Siu (2009), and extend them to include data from the 2001 and 2008-2009 recessions. See also Clark and Summers (1981), Rios-Rull (1996) and Éva Nagyál (2007) who document differences in cyclical sensitivity across age groups. Age-specific hours aggregated by efficiency-weighting constituent groups, following the procedure discussed below and in the appendix, show essentially the same volatility pattern.

[7] It is important to note that what we refer to as the hours margin is an amalgam of both the intensive margin (hours per worker) and extensive margin (workers per labor force participant) that are commonly referenced in the macro-labor literature.

Table 2: Participation Margin's Share of Hours Variance

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
				A. Filtered volatility					
Cov. not incl.	20.29	10.03	8.13	6.88	5.72	10.06	32.52	**11.14**	**6.25**
Cov. incl.	28.87	16.71	13.98	12.10	10.27	16.74	39.41	**18.21**	**11.12**
				B. Cyclical volatility					
Cov. not incl.	16.59	6.02	2.95	2.73	0.61	2.88	6.75	**7.51**	**2.76**
Cov. incl.	24.91	10.75	5.57	5.19	1.20	5.45	11.90	**13.06**	**5.23**

Notes: Data from the March CPS, 1964-2010. Shown are percentage shares of total hours variation attributed to the participation margin. Total hours per age group member is the product two variables: labor force participation per age group, and hours per labor force participant in that age group. "Cov. not incl." means covariance terms are ignored, so total variation is just the sum of the variables' variances and the share attributed to the participation margin is that of labor force participation. "Cov. incl." means total variation includes covariance terms, so total variation is the sum of the variables' variances plus two times their covariance; the share attributed to the participation margin is the variance of labor force participation plus the covariance, divided by total variation. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures.

Following Hansen (1985), the variance of hours is decomposed as:

$$Var(hpc) = Var(hplf) + Var(lfpr) + 2Cov(hplf, lfpr)$$

for hours per capita hpc, hours per labor force participant $hplf$, and the labor force participation rate $lfpr$. In Table 2 we present this decomposition of the variance of hours worked into these two margins, using HP-filtered log data. This table shows the proportion of hours variation by age group that can be attributed to the participation margin. We focus our discussion on the proportion of cyclical variance of hpc owing to the participation margin, presented in Panel B.[8] With covariance terms not included, the participation margin explains less than one fifth of the variation of any age group. For the 15-29 year old age group as a whole, the participation margin accounts for only 8% of hours fluctuations. For teenagers this is higher at 17%; nonetheless, more than four fifths of the variance of their hours worked is due to the hours margin. The bulk of all age groups' cyclical hours variation is due to variation in hours per labor force member.

The second row of Panel B presents an alternative decomposition which accounts for the covariance between hours per labor force member and labor force members per capita. Specifically,

[8] Again, this is calculated as a projection on a constant, current detrended aggregate output, and current and lagged detrended aggregate hours. The interpretation of results presented in Panel A on the filtered variance is essentially the same.

the participation margin's share is now defined as its variance plus the covariance, divided by the total variance of hours worked. Both rows of Panel B give the same message (as indeed does Panel A and its results for filtered volatilities). With the inclusion of covariance terms, the participation margin accounts for only 13% of hours variation for the young (15-29 year old) age group, and only 5% for the old.

Hence, fluctuations in hours per labor force participant account for the bulk of hours variation for all age groups.[9] Consequently, it does not appear that explanations centered on differences in the cyclicality of participation are of first-order importance for generating greater volatility of young hours over the business cycle.

B Wages

From the March CPS, we use information on labor income and hours worked to construct annual series for hourly wages over the years 1963-2009.[10] These wages are then deflated by the GDP deflator to obtain real wages. Given our interest in wage cyclicality, we construct wage rates in a manner mitigating composition effects that stem from labor heterogeneity. Specifically, we classify individuals into 220 highly disaggregated demographic groups, and weight observations to derive efficiency measures of age-specific labor input. Our procedure is an extension of that used by Lawrence F. Katz and Kevin M. Murphy (1992) and Krusell et al. (2000) and is detailed in the Online Appendix.[11] We then HP-filter these series to isolate fluctuations at the business cycle frequency.

The first row in Table 3 reports the percent standard deviation of the HP-filtered hourly real wage rates by age. We see a decreasing pattern in volatility by age with an upturn in the 60-64 age group. Row 3 presents the percent standard deviation of the *cyclical* age-specific series. As in Row 1, we see the familiar decreasing pattern of volatility by age, with a slight upturn at the end of the age distribution. We see that the real wage of 15-29 year olds is about one and half times as volatile as that of the 30-64 age group.

[9] Though not reported here, we also performed a decomposition exercise of the volatility of hours per labor force participant into its two components: the intensive and extensive margins. We find that the relative contributions of each margin are very similar for all age groups. Namely, between two-thirds and three-quarters of the hours margin variation is due to the extensive margin (workers per labor force participant). Hence, as has been found in the aggregate, hours variation for all age groups is accounted for largely by movements in and out of employment.

[10] These data are taken from the March CPS questions pertaining to "last year." Hence the surveys from 1964-2010 provide data for the years 1963-2009, which we of course take into account when deflating and constructing cyclical measures.

[11] Using weekly wages, as in Katz and Murphy (1992), yields similar results to those we report here for hourly wages.

Table 3: Volatility of Real Hourly Wages by Age Group

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	2.57	2.14	1.65	1.21	1.38	1.12	2.12	**1.64**	**1.16**
R^2	0.15	0.24	0.17	0.18	0.19	0.29	0.15	**0.29**	**0.26**
cyclical volatility	1.01	1.06	0.69	0.51	0.60	0.61	0.82	**0.88**	**0.59**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported.

To summarize, we identify two key findings regarding labor market differences between young and old workers. First, young workers experience hours worked volatility that is almost twice that of old workers. Second, real wage volatility of young workers is about one and a half times that of old workers. In the following sections, we explore the implications of these facts for real business cycle analysis. We refer to these two results as our *labor market facts*.

C Robustness checks

The Online Appendix reports further results showing that both the age-specific wage and hours volatility patterns hold, after conditioning upon several demographic characteristics. We briefly comment on the results here.

We first investigate differences across educational attainment. Tables OA1-OA2 represent the analogs of Tables 1 and 3 for those with less education (high school diploma and less), while Tables OA3-OA4 report the results for those with more education (more than high school diploma). Within each education group, the young exhibit greater volatility of both hours and wages relative to the old.

Tables OA5-OA6 represent the analogs of Tables 1 and 3 for males, while Tables OA7-OA8 report the results for females. Within each gender group, the young exhibit greater volatility of both hours and wages relative to the old.

As such, our labor market facts are robust at these finer levels of aggregation. Moreover, this indicates that age (or equivalently, labor market experience) is not simply a proxy for other demographic characteristics in terms of hours and wage volatility.[12]

[12] See also Gomme et al. (2004) who provide similar analysis with respect to hours along the dimensions of marital status and industry of employment.

III Labor Supply Channels

In this section, we demonstrate in a very general class of models that differences in labor supply characteristics alone cannot explain the two facts regarding age-specific labor market fluctuations documented above.

To focus attention on labor supply differences, throughout this section we assume that labor demand is symmetric across young and old. Specifically, consider an economy where production is summarized by an aggregate function $Y = F(A, K, H_Y, H_O)$. Here A denotes aggregate productivity/technology, K capital input, H_Y labor input of young workers, and H_O labor input for all other (i.e., old) workers. We assume that the marginal products with respect to capital and both labor inputs are positive and diminishing.

Throughout this paper, we assume profit maximization and price-taking by the representative firm. This implies that the wage rates for young and old labor, W_Y and W_O, respectively, are given by:

$$\begin{aligned} W_Y &= F_{H_Y} \\ W_O &= F_{H_O} \end{aligned}$$

Denoting the log-linearized value of a variable with a circumflex, we get:

$$\hat{W}_Y = \eta_{Y,A}\hat{A} + \eta_{Y,K}\hat{K} + \eta_{Y,Y}\hat{H}_Y + \eta_{Y,O}\hat{H}_O \tag{1}$$

$$\hat{W}_O = \eta_{O,A}\hat{A} + \eta_{O,K}\hat{K} + \eta_{O,Y}\hat{H}_Y + \eta_{O,O}\hat{H}_O. \tag{2}$$

Here $\eta_{x,z}$ is the elasticity of the marginal product of x with respect to z, $\eta_{x,z} = \frac{F_{x,z} \times z}{F_x}$; for the sake of exposition, we denote $\eta_{H_Y,z}$ as simply $\eta_{Y,z}$, and similarly for H_O.[13]

We define a production function, $F(.)$, as being *symmetric in cyclical labor demand characteristics* if it has the following properties:

1. $\eta_{Y,A} = \eta_{O,A}$
2. $\eta_{Y,K} = \eta_{O,K}$
3. $(\eta_{O,Y} - \eta_{Y,Y}) = (\eta_{Y,O} - \eta_{O,O}) = x$
4. $x \geq 0$

The first condition says that the elasticity of the marginal product of both young and old labor input to a technology shock are equal. The second says the same thing with respect to capital.

[13] The characterization of log-linear responses is standard in the literature (where models are typically solved for log-linearized dynamics about steady state), and is informative regarding the cyclical properties of model for deviations of "business cycle" magnitude.

The third is a natural symmetry condition on the elasticities of the marginal products of labor. For instance, any production function that features constant returns to scale in K, H_Y, H_O and satisfies condition (2), also trivially satisfies condition (3).[14]

The final condition is a natural sign restriction. Given diminishing marginal products, $\eta_{Y,Y}, \eta_{O,O} < 0$, then young and old labor inputs being complements, $\eta_{O,Y}, \eta_{Y,O} > 0$, is a sufficient (but not necessary) condition for the sign restriction to be satisfied. In the case where labor inputs are substitutes, $\eta_{O,Y}, \eta_{Y,O} < 0$, condition (4) states a natural requirement: That the marginal product of H_Y is more diminishing in H_Y than is the marginal product of H_O with respect to H_Y, and vice versa.[15]

In all of our propositions, we focus on results that deliver non-negative comovement of hours and wages for young workers, as we find in the data.[16] All proofs are contained in the Appendix. Our first proposition does not require imposing any conditions on the characteristics of labor supply. Hence, Proposition 1 holds allowing for arbitrary differences in the cyclical properties of labor supply between young and old.

Proposition 1 *If the production function satisfies conditions (1) - (4), then for any specification of young and old labor supply, it is impossible for the response of young hours and young wages to a business cycle shock to be greater than for the old.*

The intuition for this result is straightforward. Consider the standard textbook treatment of the labor market, where labor supply (demand) is an upward (downward) sloping function of the wage. Procyclicality of hours and wages requires a business cycle shock that shifts the labor demand schedule. Symmetry in labor demand characteristics implies that the labor demand curves for young and old labor: (a) have the same slope, and (b) shift by the same amount in response to the cyclical shock.[17]

In this case, there is only one way to generate a greater response of young hours to the shock relative to old hours: The young labor supply curve must be more elastic (after both income and substitution effects are taken into account). That is, the young labor supply curve must be flatter.

[14]See the Lemma in the Appendix.

[15]Note that the production functions used in essentially all macroeconomic models satisfy these symmetry conditions. As examples, this includes the class of Cobb-Douglas functions, $Y = AK^{\alpha}H_Y^{\gamma}H_O^{\phi}$, and CES functions $Y = AK^{\alpha}\left[\gamma H_Y + (1-\gamma)H_O\right]^{1-\phi}$, regardless of returns to scale.

[16]In the data, the correlation of cyclical hours and wages for the 15-29 yr old age group is 0.202. Note that this correlation is weaker than the correlation of age-specific hours to either aggregate hours or output. See also Mark Bils (1989) for further evidence on the procyclicality of real wages.

[17]It is precisely these restrictions that are removed when we move to the capital-experience complementarity model in Section IV.

But this immediately implies that the response of the young wage must be smaller than the response of the old wage.

Therefore, it is not possible for a production function with symmetry in cyclical labor demand characteristics to yield a larger response of both young hours *and* wages, irrespective of how one specifies labor supply characteristics.[18] We view this as an instructive result. It is well known that standard RBC models embody weak endogenous propagation mechanisms (see, for instance, Rebelo and King, 1999). As a result, the volatility properties of endogenous variables in such models are determined almost exclusively by their responsiveness to exogenous shocks. Proposition 1 indicates, for instance, that upon impact of a technology shock (i.e., a deviation in $\hat{A}$), it is impossible for $\hat{H}_Y > \hat{H}_O$ and $\hat{W}_Y > \hat{W}_O$. Moreover, the proposition indicates that this must also be true along any steady-state transition path – for instance, in response to $\hat{K}$ deviations induced by the dynamic response to a business cycle shock.

While instructive, Proposition 1 is not sufficient to characterize the variances of hours and wages. This can be seen in the proposition's proof: Determining the relative magnitude of variances requires the signing of covariance terms as well. In general, this cannot be done without putting more structure on the characteristics of labor supply. Our second proposition, however, establishes a special case where this can be done.

Proposition 2 *Suppose the production function satisfies conditions (1) - (4). If the model is summarized by only one state variable, then for any specification of young and old labor supply, it is impossible to match the labor market facts.*

The key to Proposition 2 is that all covariances can be traced back to the variance of the single state variable, and other variables' response to that state variable.

In order to provide results for models with multiple state variables, we must make some assumptions regarding labor supply, i.e. the specification of the household side of the model. In what follows we assume that young and old workers live in perpetuity and belong to the same representative household.[19] The unified household construct allows us to restrict differences in the "wealth effect" on labor supply.

[18] Indeed, this is true even without symmetry conditions (3) and (4); all that is required is the weaker condition, that $(\eta_{O,Y} - \eta_{Y,Y}) \geq (\eta_{Y,O} - \eta_{O,O})$, and Proposition 1 holds.

[19] Note that this can be viewed as a simple specification where, in every period, some young workers are "born," some young workers age and become old workers, and some old workers "die," in such a way as to maintain a constant share of young and old workers. This representation serves our purposes, since we are interested only in deriving workers' labor supply functions.

Our final result in this section is contained in Proposition 3. The key restriction is that the wealth effect on labor supply be equated across young and old agents.[20] Our proposition holds for any type of time-separable preferences used in the business cycle literature. These include the commonly-used "balanced growth" preferences of King, Plosser and Rebelo (1988) that features separability between consumption and hours, as well as the non-separable preferences of Greenwood, Hercowitz and Huffman (1988) that feature "zero wealth effect" on labor supply.

Proposition 3 *Let preferences for young and old workers be given by* $U(C_Y, H_Y)$ *and* $V(C_O, H_O)$*, respectively. Suppose* U, V *satisfy the usual regularity conditions (specifically,* U, V *decreasing and weakly convex in hours) and have identical wealth effects on labor supply. If the production function satisfies conditions (1) - (4), then it is impossible to match the labor market facts.*

Hence, regardless of the age differences embodied in the utility functions U and V, it is impossible to simultaneously generate greater volatility of hours and wages of the young relative to the old when the wages and hours of the young positively covary; this is true when wealth effects are identical within the household. Thus, for a broad class of preferences, a model featuring symmetric labor demand characteristics cannot explain the labor market facts presented in Section II.

IV A Model with Age-Specific Labor Demand

Here, we present a model featuring age-specific differences in the characteristics of labor demand to rationalize the labor market facts presented in Section II. The remaining features of the model – in particular, household preferences – are specified to conform as closely as possible to the standard RBC model. This allows us to focus on the role of age differences in labor demand.

We view our model of capital-experience complementarity as speaking to complementarities in production between experienced labor and factors that are in fixed short-run supply to the firm. These factors may include organizational capital, firm-specific capital, firm know-how, or operational/procedural knowledge that inherently requires (or is embodied in) experienced labor. Since this type of knowledge or capital is hard to adjust in the short-run, it is natural that cyclical fluctuations in output result in greater variation of young, inexperienced labor that is less tied

[20] In the Online Appendix, we consider models that allow for *different* wealth effects on labor supply across agents. In such cases, we cannot analytically rule out the possibility of greater volatility of hours and wages for the young relative to the old. However, in a numerical exercise, we find that models allowing for such differences cannot come close, *quantitatively*, to reconciling our two labor market facts.

to these factors, while demand for old, experienced labor exhibits behavior that resembles labor hoarding.[21]

A Households

The economy is populated by a large number of identical, infinitely-lived households. Each household is composed of a unit mass of family members. For simplicity, we assume there are only two types of family members, *young* and *old*, and let s_Y denote the share of family members that are young.[22] Family members derive instantaneous utility from consumption C_i and disutility from hours spent working N_i, according to $U_i\left(C_i, N_i\right)$, where $i \in \{Y, O\}$ denotes either young or old.

The representative household's date t problem is to maximize

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\left[s_Y U_Y\left(C_{Y\tau}, N_{Y\tau}\right) + (1-s_Y) U_O\left(C_{O\tau}, N_{O\tau}\right)\right], \tag{3}$$

subject to

$$s_Y C_{Y\tau} + (1-s_Y) C_{O\tau} + K_{\tau+1} = (1-\delta) K_\tau + r_\tau K_\tau + s_Y W_{Y\tau} N_{Y\tau} + (1-s_Y) W_{O\tau} N_{O\tau}, \ \forall \tau \geq t, \tag{4}$$

with $0 < \beta < 1$, $0 \leq \delta \leq 1$. Here K_t denotes capital holdings at date t, r_t is the rental rate, W_{Yt} is the wage rate of young workers, and W_{Ot} is the wage rate of old workers. The household takes all prices as given.

We specify the instantaneous utility functions to be:

$$U_Y = \log C_Y - \psi_Y N_Y^{1+\theta_Y} / \left(1+\theta_Y\right), \quad U_O = \log C_O - \psi_O N_O^{1+\theta_O} / \left(1+\theta_O\right).$$

The parameters $\theta_Y, \theta_O \geq 0$ govern the Frisch labor supply elasticity, while $\psi_Y, \psi_O > 0$ are used to calibrate the steady state values of N_Y and N_O. We normalize the time endowment of all family members to unity, so that $0 \leq N_{Yt}, N_{Ot} \leq 1$.

Because of additive separability in preferences, optimality entails equating consumption across all family members:

$$C_{Yt} = C_{Ot} = C_t. \tag{5}$$

[21] Of course, the measurement of factors such as organizational capital or firm know-how are very difficult. This motivates our modeling choice, as specifying complementarity between physical capital and experienced labor. The availability of high-quality data relating to these factor inputs allows us to discipline our analysis. Finally, to the extent that information technology is embodied in physical capital, we note that one might alternatively consider an environment where capital is complementary to younger workers. We view this idea, coupled with business cycle dynamics of investment in a vintage capital model as an interesting avenue for future research.

[22] Again, this can be viewed as a simplified framework in which young workers are born at a given rate x, young workers age and become old at rate x, and old workers die at rate x, so that the population shares of young and old workers remains constant.

The intertemporal first-order condition (FOC) is standard: $C_t^{-1} = \beta E_t \left[C_{t+1}^{-1}(r_{t+1} + 1 - \delta)\right]$. The FOCs for hours worked are given by $W_{Yt} = \psi_Y C_t N_{Yt}^{\theta_Y}$ for the young, and $W_{Ot} = \psi_O C_t N_{Ot}^{\theta_O}$ for the old. When $\theta_Y = \theta_O$, together with condition (5), the wealth and substitution effects on labor supply are equal for young and old workers.

B Firms

To study differences in demand for young and old labor over the business cycle, we relax two assumptions imposed on the standard RBC model's production technology. First, we allow hours of young, inexperienced workers and old, experienced workers to be distinct factor inputs. Second, we drop the Cobb-Douglas assumption of unit elasticity of substitution across inputs, and consider a nested CES functional form. In all of our analysis, we assume that production is constant returns to scale, and that final goods are produced by perfectly competitive firms.

We consider the following production function specification:

$$Y_t = \left[\mu \left(A_t H_{Yt}\right)^{\sigma} + (1-\mu)\left[\lambda K_t^{\rho} + (1-\lambda)\left(A_t H_{Ot}\right)^{\rho}\right]^{\sigma/\rho}\right]^{1/\sigma}, \quad \sigma, \rho < 1. \tag{6}$$

Labor-augmenting technology follows a deterministic growth trend with stationary shocks: $A_t = \exp\left(gt + z_t\right)$ where $z_t = \phi z_{t-1} + \varepsilon_t$ and $\phi \in (0,1)$, $E\left(\varepsilon\right) = 0$, $0 \leq Var\left(\varepsilon\right) = \sigma_{\varepsilon}^2 < \infty$, and $g > 0$ is the deterministic growth rate of technology. Since technology augments both H_O and H_Y, and given the households' preferences, the economy exhibits balanced growth.

Profit maximization on the part of the firm entails equating factor prices with marginal revenue products. The FOCs are:

$$r_t = Y_t^{1-\sigma}(1-\mu)\Omega_t \lambda K_t^{\rho-1}, \tag{7}$$

$$W_{Ot} = Y_t^{1-\sigma}(1-\mu)\Omega_t(1-\lambda)A_t^{\rho} H_{Ot}^{\rho-1}, \tag{8}$$

$$W_{Yt} = Y_t^{1-\sigma}\mu A_t^{\sigma} H_{Yt}^{\sigma-1}, \tag{9}$$

where $\Omega_t \equiv \left[\lambda K_t^{\rho} + (1-\lambda)\left(A_t H_{Ot}\right)^{\rho}\right]^{(\sigma-\rho)/\rho}$.[23]

The degree of diminishing marginal product differs between young and old labor whenever $\sigma \neq \rho$. The elasticity of substitution between old workers and capital is given by $(1-\rho)^{-1}$, while the elasticity of substitution between young workers and the H_O-K composite is $(1-\sigma)^{-1}$. Adapting

[23]Krusell et al. (2000) consider a similar production specification in their analysis of trends in relative wages. They distinguish labor skill by the level of educational attainment (as opposed to experience in our model). Castro and Coen-Pirani (2008) use the same production function as Krusell et al. (2000), and consider changes in the elasticities of substitution to explain changes in the relative volatility of hours for high educated workers since 1984.

the terminology of Krusell et al. (2000), we define production as exhibiting *capital-experience complementarity* if $\sigma > \rho$, when we equate age with labor market experience.

To see how such a production technology can generate greater volatility of young labor input relative to the old, consider a simple example. Suppose that old labor is a perfect complement to capital (i.e., $\rho \to -\infty$), while young labor is not ($\sigma > \rho$). Since capital is in inelastic supply in the short-run, a productivity shock generates no response in the quantity of old labor hired; the only variation is in the quantity of young labor.

C Equilibrium

Equilibrium is defined as follows. Given $K_0 > 0$ and the stochastic process for technology, a *competitive equilibrium* is an allocation, $\{C_t,\ N_{Yt},\ N_{Ot},\ K_t,\ Y_t,\ H_{Yt},\ H_{Ot}\}$, and price system, $\{W_{Yt},\ W_{Ot},\ r_t\}$, such that: given prices, the allocation solves both the representative household's problem and the representative firm's problem for all t; the capital rental market clears for all t; and labor markets clear ($H_{Yt} = s_Y N_{Yt};\ H_{Ot} = (1 - s_Y)\, N_{Ot}$) for all t. Walras' law ensures clearing in the final goods market: $C_t + K_{t+1} = Y_t + (1 - \delta)\, K_t$, for all t. Finally, for the purposes of model evaluation, we define aggregate hours worked as $H_t = H_{Yt} + H_{Ot}$.

V Quantitative Specification

In this section, we describe the quantitative specification of our model. To maintain comparability with the RBC literature, we perform a standard calibration when possible. However, the parameters governing elasticities of substitution in production cannot be calibrated to match first moments in the U.S. data. Instead, we adopt a structural, instrumental variables estimation procedure to identify these values using data from the Bureau of Economic Analysis (NIPA) and Census (CPS). After describing the procedure, we discuss calibration of the remaining parameter values. Recalling the empirical results of Section II, we identify young and old workers in the model with 15-29 and 30-64 year old age groups, respectively, in the data.[24]

[24] This is clearly an extreme specification, in that the distinction between young and old labor input is determined by a single age threshold. However, we have found in our estimation results are robust to five year variation in this cut-off age.

A Estimation

To estimate the elasticity parameters σ and ρ, consider the factor demand equations implied by our model.[25] The firm's FOC with respect to H_{Yt} (9) can be logged and first-differenced into

$$\Delta \log W_{Yt} = \alpha_0 + (\sigma - 1)\Delta \log (H_{Yt}) + (1 - \sigma)\Delta \log (Y_t) + \sigma u_t, \tag{10}$$

where α_0 is a constant, and u_t is a function of current and lagged shock innovations,

$$u_t = \varepsilon_t - (1 - \phi)\left(\varepsilon_{t-1} + \phi\varepsilon_{t-2} + \phi^2\varepsilon_{t-3} + ...\right).$$

Hence equation (10) represents a textbook labor demand equation.

Therefore our strategy for estimating the elasticity parameter σ amounts to estimating the responsiveness of the young labor demand relation. Empirical identification is obtained from the response of W_Y to (exogenous) changes in H_Y and Y in the aggregate data. Abstracting from endogeneity issues (which we address below), σ could be estimated from a simple regression.

The age-specific wages analyzed in Section II are constructed using hours data in order to translate direct information on labor income in the CPS into measured wages. It is possible that there is error in our measurement of hours, in spite of the aggregation across individuals within each age group. This would contaminate our measurement of wages and induce unnecessary imprecision into our IV estimates. A simple fix is to estimate a variant of (10) relating hours directly to labor income:

$$\Delta \log LI_{Yt} - \Delta \log Y_t = \alpha_1 + \sigma\left(\Delta \log H_{Yt} - \Delta \log Y_t\right) + \sigma u_t \tag{11}$$

where $LI_{Yt} \equiv W_{Yt}H_{Yt}$ denotes labor income earned by young workers. Again, direct measures of LI_{Yt} and H_{Yt} can be obtained from the CPS, while Y_t is available from the NIPA.

To estimate ρ we proceed in a similar manner. The firm's FOCs with respect to K_t and H_{Ot}, (7) and (8), represent factor demand equations for old labor and capital. Since these are of the same functional form, they can be combined to obtain an equation that depends on the slope parameter, ρ, alone. In logged, first-differenced form:

$$\Delta \log W_{Ot} - \Delta \log r_t = \alpha_2 + (\rho - 1)\left(\Delta \log H_{Ot} - \Delta \log K_t\right) + \rho u_t.$$

Again, we can avoid unnecessary imprecision by estimating the following version with instrumental variables:

$$\Delta \log Q_{Ot} - \Delta \log Q_{Kt} = \alpha_2 + \rho\left(\Delta \log H_{Ot} - \Delta \log K_t\right) + \rho u_t. \tag{12}$$

[25] A similar approach is used in Burnside, Eichenbaum and Rebelo (1995) and the references therein.

Here, Q_{Ot} denotes the share of national income earned by old labor, and Q_{Kt} the share of national income earned by capital. Identification of ρ is obtained from the response of national income shares to (exogenous) short-run variation in the factor input share.

Importantly, our procedure does not require imposing any restrictions from the model's specification of household behavior.[26] The only assumptions required to pin down σ and ρ are: (i) profit maximization on the part of firms, and (ii) that changes in factor prices reflect changes in marginal revenue products. As is obvious from our estimating equations, (11) and (12), identification does not rely upon the fact that young hours are more volatile over the cycle than old hours. Moreover, no aspect of our approach imposes that $\sigma > \rho$. Whether or not this is satisfied depends on the relation between aggregate prices and quantities observed in the data.

Endogeneneity Since our empirical equations are based on factor demand equations we must address the potential endogeneity of the regressors. The structural equations identify the error term as due to shocks to technology. To obtain unbiased estimates, we isolate variation in our regressors that is unrelated to shocks shifting firms' factor demand, be they technology shocks or other omitted factors from the FOCs.

We do so by adopting an instrumental variables approach using lagged birth rates. Intuitively, these instruments allow us to identify changes in current labor supply – due to changes in past fertility – that are uncorrelated to shifts in factor demand.[27] Recall that:

$$u_t = \varepsilon_t - (1-\phi)\left(\varepsilon_{t-1} + \phi\varepsilon_{t-2} + \phi^2\varepsilon_{t-3} + ...\right).$$

Lagged birth rates are *valid* if fertility is exogenous to past technology shock innovations, $\{\varepsilon_{t-j}\}_{\forall j>0}$. If one believes that fertility decisions, say, 15 years ago might be endogenous to innovations at least 15 years ago, then some bias might be induced. However, note that in the case of the 15-year lagged birth rate, the concern is its correlation with the sum $(1-\phi)\sum_{j=14}^{\infty}\phi^j\varepsilon_{t-j-1}$ in u_t. For standard values of shock persistence, ϕ, relevant for our analysis, this impact is almost negligible. Obviously, for birthrates of larger lag, this is even smaller. We thus conclude that, from an empirical standpoint, lagged birth rates are valid instruments.[28]

[26] We see this as a virtue since our goal is to study the quantitative role of differences in the cyclical demand for young and old labor.

[27] See also Paul Beaudry and David Green (2003) who use exogenous demographic variation as an instrument in production function estimation.

[28] See the Online Appendix for further detail.

Table 4: ESTIMATION RESULTS

	Point Estimate	Standard Error	First Stage F-Statistic	J-test p-value	$\sigma = \rho$ p-value
σ	0.662	0.048	10.829†	0.425	< 0.001
ρ	0.201	0.016	13.891†		

Notes: Data from the March CPS, 1964-2010. Estimation is two-step GMM with Andrews's (1991) HAC standard errors with bandwidth chosen by Newey and West (1994), using lagged birth rates as instruments. J denote's Hansen's J-test; $\sigma = \rho$ denotes the F-test of the null that the two parameters are equal. † indicates significance at the 5%-level for Stock and Yogo's (2002) TSLS weak instrument tests based on 0.2 maximal size distortion or 0.1 maximal bias.

Results Our theory suggests that the error terms u_t are correlated. Therefore, joint estimation is efficient. We use a system approach to estimate equations (11) and (12) by two-step GMM following Lars P. Hansen (1982). Heteroskedasticity and autocorrelation robust standard errors are estimated following Donald W. K. Andrews (1991) with optimal bandwidth chosen according to Whitney K. Newey and Kenneth D. West (1994).[29] This ensures that the standard errors we report and use for hypothesis tests account for any serial correlation or heteroskedasticity that is present in u_t.

We must use instruments to consistently estimate σ and ρ, and must ensure our instruments are not only valid but also *relevant* and *robust* to weak instrument issues. Hence, we test whether any possible instrument weakness leads to biased estimates or distorted hypothesis tests. Using James H. Stock and Motohiro Yogo's (2005) critical values for the first stage F-statistic, we reject at the 5%-level the hypothesis that weak instruments lead to bias in our estimates or distortions of our hypothesis tests' size.[30] In other words, we can reject that our instruments' weakness distorts the point estimates or hypothesis tests we report.

Table 4 presents the results of the estimation. The point estimate of $\rho = 0.201$ indicates that the elasticity of substitution is a little more than unity in K and H_O; in contrast, the estimate of $\sigma = 0.662$ indicates that the substitution elasticity is substantially larger between H_Y and the K-H_O composite. Conducting the F-test of the null hypothesis that $\sigma = \rho$, we obtain an extremely low p-value suggesting that the difference between σ and ρ is statistically significant at the 0.1%

[29] In the Online Appendix we show that our estimates are robust to alternative GMM specifications.

[30] In particular, we test the null hypotheses that point estimates are biased by larger than 0.1 of the true value, and that hypothesis tests have size distortions in excess of 0.2. Note that in our framework with one endogenous regressor, the first stage F-statistic is John G. Cragg and Stephen G. Donald's (1993) statistic.

level. Moreover, the difference is in the "right" direction for the interpretation of capital-experience complementarity ($\sigma > \rho$).[31] In summary, our results demonstrate strong instruments, precisely estimated parameters, robustness across a variety of specifications and a statistically significant difference in the elasticity of substitution between young or prime-aged hours and capital.

B Calibration

The remaining parameters are calibrated in the standard manner. We set $\beta = 0.99$ and $\delta = 0.025$ to correspond to quarterly time periods. The values of s_Y, ψ_Y, and ψ_O are set to match the average values of the 15-29 year old population shares, and fractions of time spent in market activities by young and old individuals observed in postwar U.S. data. Since θ_Y and θ_O govern elasticities, we cannot calibrate these to match first moments. Moreover, microeconomic estimates do not necessarily correspond to the representative household's labor supply elasticity, as noted by Rogerson (1988) and others. As such, we consider various values to illustrate the quantitative properties of our models.

Following Krusell et al. (2000), we calibrate the share parameters in production, μ and λ, to match national income shares. Specifically, given the estimated values for σ and ρ, we set μ and λ to match the 1964-2010 national income shares of $Q_K = 0.37$ and $Q_O = 0.50$.

With values for $\{\hat{\sigma}, \hat{\rho}, \mu, \lambda\}$ we back out the implied technology series $\{A_t\}$ using data on output and factor inputs. From $\{A_t\}$, we obtain a quarterly estimate of $\hat{\phi} = 0.94$ and $\hat{\sigma}_\varepsilon = 0.0064$.

VI Quantitative Evaluation

In this section, we evaluate the quantitative predictions of the capital-experience complementarity model. Specifically, we study the performance of the model with respect to our two new empirical facts – the volatility of hours and wages of the young relative to the old.

Column I in Table 5 presents business cycle statistics for HP-filtered U.S. data. As discussed in Section II, the volatility of young hours worked is greater than that of old hours worked, with a relative standard deviation of 1.85. As before, the young are defined as those aged 15-29 year olds, and the old as those 30-64 years old. Relative to aggregate output, young hours exhibits greater cyclical volatility, while old hours exhibits somewhat lower volatility. The volatility of aggregate

[31] Compared to Krusell et al. (2000), who differentiate on skilled/unskilled versus experienced/inexperienced labor, our estimated σ is similar (0.66 versus their 0.40) but our ρ is different (0.2 versus their -0.5). This means that Krusell et al. (2000) find that capital and skilled labor are more complementary than we find capital and experienced labor to be. This is an interesting distinction for future research to explore.

Table 5: Data and Model Moments

	Data	Model		
	(I)	(II)	(III)	(IV)
$sd(H_Y)/sd(H_O)$	1.85	2.07	1.85	1.49
$sd(H_Y)/sd(Y)$	1.64	1.60	1.46	1.20
$sd(H_O)/sd(Y)$	0.89	0.77	0.79	0.81
$sd(H)/sd(Y)$	1.10	1.04	1.01	0.94
$sd(W_Y)/sd(W_O)$	1.50	1.00	1.17	1.50
$sd(W_Y)/sd(Y)$	0.47	0.26	0.31	0.41
$sd(W_O)/sd(Y)$	0.31	0.26	0.27	0.27
$sd(Y)$	1.56	1.32	1.28	1.21
$sd(Y)/sd(z)$	-	1.58	1.53	1.45
θ_Y	-	0	0.04	0.14
θ_O	-	0	0	0
Target	-	-	$\frac{sd(H_Y)}{sd(H_O)}$	$\frac{sd(W_Y)}{sd(W_O)}$

Notes: Column I are sample moments calculated from HP filtered data from March CPS, 1964-2010. Columns II-IV are sample moments calculated from model-simulated data. The row Target indicates what moment is targeted by the Frisch elasticity parameter.

hours is of a similar magnitude to that of aggregate output; in fact, in our data, which includes the onset of the Great Recession, the ratio of standard deviations of aggregate hours to output exceeds unity.

The remaining rows in Column I report volatility statistics for real wages for the two age groups. As noted in Section B, the volatility of wages is also greater for the young than for the old. For our two age groups, the ratio of real wage volatility is 1.50.[32] Since our model embodies a standard inter-temporal Euler condition, the model's predictions for consumption and investment are unchanged relative to a standard RBC model; as such, we do not report them here.

We begin with examination of the capital-experience complementarity model where we set $\theta_Y = \theta_O = 0$, so that utility is linear in labor. This is a useful benchmark since the standard RBC model (with homogenous labor and Cobb-Douglas production function) requires very high aggregate labor supply elasticity to generate significant volatility of hours worked; the indivisible labor model (with perfectly elastic labor supply) generates a ratio of the standard deviation of

[32] We report the volatility for cyclical fluctuations in hours and real wages, as constructed in Section II. Given the focus on business cycle fluctuations in hours and wages, we concentrate on the variation that is due to the cycle.

hours to output of approximately $0.7 - 0.75$.[33]

As Column II of Table 5 reports, the capital-experience complementarity model generates significant volatility of young hours relative to old hours. The ratio of standard deviations is 2.07, which is 12% greater than that observed in the US data. The model also has no difficulty in generating young hours that are more volatile than output, and old hours that are less volatile. For both statistics, the model slightly understates these relative volatilities: the standard deviation of young hours to output in the model is 1.60 compared to 1.64 in the data, and the standard deviation of old hours to output in the model is 0.77 compared to 0.89 in the data.[34]

As a by-product of this success, the model also generates significant volatility of aggregate hours. In fact, the relative volatility of aggregate hours to output is very close to that observed in the data. The relative standard deviation is 1.04 in the model, whereas it is 1.10 in the data. In this sense, the capital-experience complementarity model represents a potential resolution to the RBC literature's inability to generate sufficient hours worked volatility.

Finally, we note that the benchmark model generates significant volatility of aggregate output. In our model, the standard deviation of output is 1.32%, or about 84% of that observed in the US data. This is a marked improvement over the indivisible-labor version of the standard RBC model, and compares favorably with models that allow for variability in capital utilization (see Prescott, 1986; Craig Burnside and Martin Eichenbaum, 1996). Relaxing the assumption of unit elasticity of substitution in factor supplies allows the capital-experience complementarity model to generate significant endogenous amplification of productivity shocks. The relative volatility of output to the shock process is around 1.58, which is substantially larger than in the standard RBC model where this relative volatility is typically near unity (see Craig Burnside and Martin Eichenbaum, 1996).

While the benchmark calibration is surprisingly successful along the hours dimension, it cannot account for the behavior of relative wages between the young and the old. This is expected since the Frisch labor supply elasticity is infinite, and the wealth effect is identical for both young and old agents. In this case, the volatility of wages is necessarily identical. This is addressed in the next two experiments.

As already discussed, the benchmark calibration (with $\theta_Y = \theta_O = 0$) overstates the volatility of

[33] See for example, Hansen (1985), Rogerson (1988), King and Rebelo (1999).

[34] Our quantitative specification has an elasticity of substitution between capital and old hours that is close to unity ($(1-\rho)^{-1} = 1.25$), and infinite Frisch elasticity of labor supply for the old. These are the features displayed by the homogenous labor input in the standard RBC model with indivisible labor, discussed above. Thus the capital-experience complementarity model generates a relative volatility of old hours to output, $sd(H_O)/sd(Y)$, similar to the relative volatility of aggregate hours to output in the standard RBC model.

young hours relative to old hours. In Column II, we consider the following modification: we change only the labor supply elasticity of young workers to match the relative hours volatility observed in the US data. This requires increasing θ_Y from 0 to 0.04, so that the Frisch labor supply elasticity of young workers is less than that of old workers.[35] Not surprisingly, this lowers the volatility of young hours to aggregate output.

The modification also lowers the volatility of aggregate hours to output; however, the model still delivers a relative volatility that is near unity (1.01) and close to that observed in the data (1.10). Lowering the labor supply elasticity of the young also lowers the volatility of output (marginally, from 1.32 to 1.28). However, the model still embodies significant amplification, as the standard deviation of output is 53% greater than that of the exogenous shock process. Finally, we note that this modification also allows the model to match the fact that both young workers' hours and wages are more volatile than for old workers.

In Column III we consider the following modification: we change only the labor supply elasticity of the young to match the observed relative wage volatility ($sd\,(W_Y)\,/sd\,(W_O) = 1.50$). This requires increasing θ_Y to 0.14. Moreover, this modification does very well at matching the volatility of both young and old wages, relative to aggregate output, found in the US data. Not surprisingly, the lower elasticity of young labor supply induces a fall in the volatility of young hours. The relative volatility of age-specific hours ($sd\,(H_Y)\,/sd\,(H_O) = 1.49$) now understates that found in the data. Finally, we note that the model still embodies significant amplification, with $sd\,(Y)\,/sd\,(z) = 1.45$ well above unity.

In sum, we find that the capital-experience complementarity model easily captures the fact that both hours and wages of young workers are more volatile over the business cycle than for old workers.[36] That is, modeling differences in the cyclical characteristics of labor demand quantitatively accounts for our labor market facts. As a by-product of this success, the model generates volatility of aggregate hours that is very close to that of aggregate output. Finally, the model embodies significant internal amplification of business cycle shocks.

[35] We find this to be an interesting experiment since it makes clear that our results in no way rely upon assuming that young workers have greater labor supply elasticity (an assumption that, to our knowledge, has no empirical support in the literature).

[36] Because the model is driven by a single productivity shock, the model trivially generates a positive correlation between hours and wages, though this is counterfactually close to unity. As in the literature, this could easily be remedied by the inclusion of other shocks that would affect the correlation of hours and wages without affecting the model's relative volatility properties (see, for instance, Lawrence J. Christiano and Eichenbaum, 1992, and Jess Benhabib, Rogerson and Wright, 1991).

VII Conclusion

We highlight two important empirical observations regarding age differences in labor market fluctuations. First, hours worked of young workers are more volatile over the business cycle than their prime-aged counterparts. Second, real wages of the young are more volatile over the business cycle than the prime-aged.

We show that a general class of models allowing for age differences in labor supply characteristics alone cannot account for these facts. Instead, a model emphasizing age differences in labor demand factors can. Our model posits a greater complementarity of prime-aged workers' labor input with capital in production than for young workers.

Our model of capital-experience complementarity represents a minor deviation from the standard RBC model, and delivers factor demand equations that can be used to estimate structural elasticity parameters. We find that the data is consistent with capital-experience complementarity.

Quantitative evaluation of the model shows that it can easily reconcile the labor market facts. Moreover, the model obtains aggregate hours that have equal volatility to aggregate output, and embodies strong internal amplification of business cycle shocks. Altogether, the paper points to the importance of characterization of age-specific differences in the demand for labor inputs in understanding business cycle fluctuations.

Appendix

Lemma Condition (2) implies condition (3) if $F(\cdot)$ is constant returns to scale.

Lemma Proof. Since $F(\cdot)$ is homogenous of degree one in K, H_Y, H_O:

$$\begin{aligned} 0 &= F_{Y,K}K + F_{Y,O}H_O + F_{Y,Y}H_Y, \\ \frac{-F_{Y,K}K}{F_Y} &= \frac{F_{Y,O}H_O + F_{Y,Y}H_Y}{F_Y}, \\ -\eta_{Y,K} &= \eta_{Y,O} + \eta_{Y,Y}. \end{aligned}$$

Similarly, from the marginal product of old labor:

$$-\eta_{O,K} = \eta_{O,Y} + \eta_{Y,Y}.$$

Condition (2) then implies:

$$\begin{aligned} \eta_{O,O} + \eta_{O,Y} &= \eta_{Y,O} + \eta_{Y,Y}, \\ \eta_{O,Y} - \eta_{Y,Y} &= \eta_{Y,O} - \eta_{O,O}. \end{aligned}$$

which is condition 3. ■

Proposition 1 Proof. Take the difference of equations (1) and (2). Using symmetry conditions (1) and (2):

$$\begin{aligned}\hat{W}_Y - \hat{W}_O &= (\eta_{Y,Y} - \eta_{O,Y})\hat{H}_Y - (\eta_{O,O} - \eta_{Y,O})\hat{H}_O, \\ &= x(\hat{H}_O - \hat{H}_Y), \end{aligned} \tag{A1}$$

where the last equality follows from condition (3). Now impose condition (4). If $x = 0$, then the variance of young wages is identical to that of prime-age wages; this violates our requirement of a differential response of wages, irrespective of the response of hours. Now consider the case where $x > 0$. Suppose that $\hat{W}_Y - \hat{W}_O > 0$, so that the response of young wages is greater than that of old wages. Then this implies $\hat{H}_O > \hat{H}_Y$. Given our interest in $\hat{W}_Y$ and $\hat{H}_Y$ responses that are of the same sign, this violate our requirement that the response of young hours is greater than that of old hours. ■

Proposition 2 Proof. Assume there is only one state variable, S (whether it be technology, A, capital, K, or anything else). Then the model's state space representation implies that we can express the equilibrium relationships:

$$\begin{aligned}\hat{H}_Y &= B_{Y,S}\hat{S}, \\ \hat{H}_O &= B_{O,S}\hat{S}.\end{aligned}$$

Thus $Var(\ \hat{H}_Y) > Var(\hat{H}_O)$ iff $B_{Y,S} > B_{O,S}$.

Given conditions (1) - (4) characterizing symmetry in labor demand, equations (1) and (2) can be rewritten as:

$$\begin{aligned}\hat{W}_Y &= \eta_{YS}\hat{S} + \eta_{YY}B_{Y,S}\hat{S} + \eta_{YO}B_{O,S}\hat{S} = [\eta_{YS} + \eta_{YY}B_{Y,S} + \eta_{OO}B_{O,S} + xB_{O,S}]\,\hat{S}, \\ \hat{W}_O &= \eta_{YS}\hat{S} + \eta_{OY}B_{Y,S}\hat{S} + \eta_{OO}B_{O,S}\hat{S} = [\eta_{YS} + \eta_{YY}B_{Y,S} + \eta_{OO}B_{O,S} + xB_{Y,S}]\,\hat{S}.\end{aligned}$$

Then $Var(\ \hat{W}_Y) > Var(\hat{W}_O)$ iff $B_{O,S} > B_{Y,S}$. This contradicts the necessary and sufficient condition for $Var(\ \hat{H}_Y) > Var(\hat{H}_O)$. ■

Proposition 3 Proof. The FOC with respect to hours for young workers is

$$U_{H_Y}(C_Y, H_Y) = -\Lambda W_Y,$$

where Λ is the Lagrange multiplier on the household's budget constraint. Log-linearizing the FOC obtains

$$U_{H_Y C_Y} C_Y \hat{C}_Y + U_{H_Y H_Y} H_Y \hat{H}_Y = -\Lambda W_Y(\hat{\Lambda} + \hat{W}_Y),$$

which can be rewritten using $\left(\frac{U_{H_Y H_Y} H_Y}{U_{H_Y}}\right) \equiv \mathcal{U}$ as

$$\hat{W}_Y = \left[\frac{U_{H_Y C_Y} C_Y}{U_{H_Y}} \hat{C}_Y - \hat{\Lambda}\right] + \mathcal{U}\hat{H}_Y. \tag{A2}$$

Thinking of (A2) in terms of a labor supply function in $H_Y - W_Y$ space, the first term on the right hand side represents "shifts of," while the second term represents "movements along," the labor supply curve. Thus, the first term is the wealth effect while the second term is the substitution effect. Note that $\mathcal{U} \geq 0$, given that $U_{H_Y} < 0$ and $U_{H_Y H_Y} \leq 0$. For old workers with utility function $V(C_O, H_O)$, we use $\left(\frac{V_{H_O H_O} H_O}{V_{H_O}}\right) \equiv \mathcal{V}$ and derive analogously

$$\hat{W}_O = \left[\frac{V_{H_O C_O} C_O}{V_{H_O}} \hat{C}_O - \hat{\Lambda}\right] + \mathcal{V}\hat{H}_O, \tag{A3}$$

where $\mathcal{V} \geq 0$, given $V_{H_O} < 0$ and $V_{H_O H_O} \leq 0$.

Restricting the wealth effect on labor supply to be identical across agents implies that the terms in square brackets in (A2) and (A3) are equal. Using the symmetry condition (A1) implies that

$$x\left(\hat{H}_O - \hat{H}_Y\right) = \mathcal{U}\hat{H}_Y - \mathcal{V}\hat{H}_O,$$

which can be rewritten as

$$(x + \mathcal{V})\,\hat{H}_O = (x + \mathcal{U})\,\hat{H}_Y. \tag{A4}$$

Thus, for the young to have more volatile hours requires $\mathcal{V} > \mathcal{U} \geq 0$.

Using (A4), we can alternatively express (A1) as

$$\hat{W}_Y - \hat{W}_O = x\left(\frac{x + \mathcal{U}}{x + \mathcal{V}} - 1\right)\hat{H}_Y.$$

Rearranging obtains

$$\hat{W}_O = \hat{W}_Y + r\hat{H}_Y, \tag{A5}$$

where $r \equiv \frac{x(\mathcal{V} - \mathcal{U})}{x + \mathcal{V}}$. If $x = 0$, then (A5) implies that wages have equal volatility, immediately contradicting our labor market facts. Hence, it must be that $x > 0$. This implies that $r > 0$, given that we require $\mathcal{V} > \mathcal{U}$. Taking variances we get

$$Var(\hat{W}_O) = Var(\hat{W}_Y) + r^2 Var(\hat{H}_Y) + 2rCov(\hat{W}_Y, \hat{H}_Y).$$

Since the second term on the right hand side is positive and the third term is non-negative, this implies that $Var(\hat{W}_O) > Var(\hat{W}_Y)$, a contradiction of our labor market facts. Similarly, to get $Var(\hat{W}_Y) > Var(\hat{W}_O)$ we need r to be negative, or $\mathcal{U} > \mathcal{V}$; but then (A4) implies that $Var(\hat{H}_O) > Var(\hat{H}_Y)$, again contradicting our labor market facts. ■

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Online Appendix – Not for Publication

for "The Demand for Youth: Explaining Age Differences in the Volatility of Hours"

by Nir Jaimovich, Seth Pruitt and Henry E. Siu

A Hours and Wage Volatility in Further Breakdowns

Table OA1: Volatility of Hours Worked by Age Group, High School Degree and Below

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	6.07	3.02	2.84	2.09	1.78	2.02	2.90	**3.27**	**1.74**
R^2	0.82	0.74	0.69	0.72	0.62	0.64	0.18	**0.87**	**0.80**
cyclical volatility	5.48	2.60	2.36	1.78	1.40	1.61	1.23	**3.05**	**1.55**
hours share	5.90	11.18	11.62	23.54	23.56	18.89	5.30	**28.71**	**71.29**
volatility share	16.13	14.51	13.65	20.89	16.39	15.17	3.26	**44.11**	**55.89**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. Hours share is the sample average share of aggregate hours worked by the age group, reported in percentage. Volatility share is the age group's share of aggregate hours volatility, defined as the the average of age-specific cyclical volatilities weighted by hours shares, reported in percentage. For individuals whose highest educational attainment is a high school degree.

Table OA2: Volatility of Real Hourly Wages by Age Group, High School Degree and Below

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	2.78	2.24	1.88	1.48	1.38	1.04	2.52	**1.71**	**1.15**
R^2	0.15	0.14	0.03	0.16	0.12	0.19	0.03	**0.11**	**0.18**
cyclical volatility	1.09	0.84	0.33	0.59	0.47	0.45	0.42	**0.56**	**0.48**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. For individuals whose highest educational attainment is a high school degree.

Table OA3: Volatility of Hours Worked by Age Group, More than High School

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	6.38	2.50	1.92	1.53	1.43	1.74	3.34	**1.77**	**1.06**
R^2	0.28	0.38	0.22	0.50	0.21	0.22	0.03	**0.50**	**0.51**
cyclical volatility	3.41	1.54	0.91	1.08	0.65	0.81	0.59	**1.26**	**0.75**
hours share	1.34	10.49	14.78	28.73	24.82	16.13	3.71	**26.61**	**73.39**
volatility share	4.74	16.67	13.91	32.09	16.74	13.60	2.26	**37.72**	**62.28**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. Hours share is the sample average share of aggregate hours worked by the age group, reported in percentage. Volatility share is the age group's share of aggregate hours volatility, defined as the the average of age-specific cyclical volatilities weighted by hours shares, reported in percentage. For individuals whose highest educational attainment is more than a high school degree.

Table OA4: Volatility of Real Hourly Wages by Age Group, More than High School

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	4.02	2.92	1.85	1.25	1.74	1.52	2.60	**2.03**	**1.28**
R^2	0.08	0.20	0.24	0.19	0.16	0.13	0.16	**0.36**	**0.32**
cyclical volatility	1.17	1.29	0.92	0.55	0.69	0.55	1.04	**1.21**	**0.72**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. For individuals whose highest educational attainment is more than a high school degree.

Table OA5: Volatility of Hours Worked by Age Group, Males

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	6.34	2.95	2.12	1.63	1.41	1.68	2.75	**2.69**	**1.40**
R^2	0.81	0.69	0.69	0.91	0.72	0.76	0.14	**0.85**	**0.96**
cyclical volatility	5.70	2.46	1.76	1.55	1.20	1.46	1.04	**2.47**	**1.37**
hours share	3.40	10.01	13.26	26.85	24.12	17.65	4.71	**26.67**	**73.33**
volatility share	11.50	14.61	13.85	24.75	17.13	15.26	2.91	**39.64**	**60.36**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. Hours share is the sample average share of aggregate hours worked by the age group, reported in percentage. Volatility share is the age group's share of aggregate hours volatility, defined as the the average of age-specific cyclical volatilities weighted by hours shares, reported in percentage. For males.

Table OA6: Volatility of Real Hourly Wages by Age Group, Males

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	3.33	2.27	1.84	1.39	1.67	1.36	2.70	**1.72**	**1.36**
R^2	0.14	0.22	0.14	0.20	0.14	0.36	0.14	**0.23**	**0.26**
cyclical volatility	1.23	1.07	0.69	0.62	0.63	0.82	1.00	**0.83**	**0.69**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. For males.

Table OA7: Volatility of Hours Worked by Age Group, Females

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	5.44	2.09	2.06	1.40	1.27	1.46	2.44	**2.16**	**1.09**
R^2	0.64	0.59	0.45	0.72	0.46	0.51	0.04	**0.77**	**0.73**
cyclical volatility	4.35	1.60	1.38	1.19	0.86	1.04	0.51	**1.90**	**0.93**
hours share	4.26	12.16	12.91	24.69	24.22	17.48	4.29	**29.32**	**70.68**
volatility share	14.68	15.39	14.06	23.24	16.45	14.45	1.74	**45.75**	**54.25**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. Hours share is the sample average share of aggregate hours worked by the age group, reported in percentage. Volatility share is the age group's share of aggregate hours volatility, defined as the the average of age-specific cyclical volatilities weighted by hours shares, reported in percentage. For females.

Table OA8: Volatility of Real Hourly Wages by Age Group, Females

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64	**15 - 29**	**30 - 64**
filtered volatility	3.17	2.47	1.79	1.63	1.72	1.46	3.04	**1.83**	**1.36**
R^2	0.06	0.21	0.19	0.19	0.23	0.09	0.14	**0.29**	**0.32**
cyclical volatility	0.75	1.13	0.78	0.71	0.83	0.45	1.12	**0.98**	**0.77**

Notes: Data from the March CPS, 1964-2010. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. For females.

B Different Wealth Effects

Qualitatively, the twin facts are that young hours and wages are more cyclically volatile than old hours and wages; *quantitatively*, the differences are large as the standard deviation of young hours is 1.9 times that of prime-aged hours, and the standard deviation of young wages is 1.5 times that of than prime-aged wages.

The paper proves that differences in the young and old labor supply characteristics alone cannot deliver the twin facts, when wealth effects for the young and old are the same. In this section we show how when we allow for the presence of different wealth effects across young and old then we cannot rule out its ability to match the twin facts. Specifically, we begin with an example that shows how the twin facts can qualitatively be delivered *only if* the workers with a "stronger" wealth effect also have a "weaker" substitution effect. We then discuss the basic economic forces behind this result via simple graphical analysis. We conclude by confirming this intuition via numerical results and show that even in cases where the model can qualitatively match the twin facts, it cannot do it quantitatively.

Example

Let young and old have different wealth effects. In this case there is a necessary condition such that we cannot rule out the possibility of accounting for the twin facts.

Specifically, suppose preferences for the young are separable as in King, Plosser and Rebelo (1988) (KPR)

$$U_Y = \log(C_Y) - \psi_Y N_Y^{1+\theta y}$$

while the preferences for the old are as in Greenwood, Hercowitz and Huffman (1988) (GHH)

$$U_O = \log(C_O - \psi_O N_O^{1+\theta_O}).$$

Note that in this case the young have a stronger wealth effect then the old (who have zero wealth effect).

The representative household's problem is given in equations (3) and (4), leading to the following two log-linear first order conditions for the supply of hours by the young and the old

$$\begin{aligned} \theta_Y \hat{H}_Y &= \hat{W}_Y - \hat{C}_Y \\ \theta_O \hat{H}_O &= \hat{W}_O \end{aligned}$$

Substituting these two equations into equation (A1) we get

$$(x + \theta_Y)\, \hat{H}_Y + \hat{C}_Y = (x + \theta_O)\, \hat{H}_O$$

and rearranging the last equation

$$\frac{x + \theta_Y}{x + \theta_O} \hat{H}_Y + \frac{1}{(x + \theta_O)} \hat{C}_Y = \hat{H}_O \tag{OA1}$$

From equation (OA1) it follows that

$$\left[\frac{x + \theta_Y}{x + \theta_O}\right]^2 Var(\hat{H}_Y) + \left(\frac{1}{x + \theta_O}\right)^2 Var(\hat{C}_Y) + 2\frac{x + \theta_Y}{(x + \theta_O)^2} Cov(\hat{H}_Y, \hat{C}_Y) = Var(\hat{H}_O) \tag{OA2}$$

Equation (OA2) shows that as long as we restrict ourselves to the case where $Cov(\hat{C}_Y, \hat{H}_Y) \geq 0$ then a necessary condition to match the hours fact is that $\theta_O > \theta_Y$. That is, in the situation where the young have a stronger wealth effect, the old must have a stronger substitution effect. Similarly, substituting equation (OA1) to equation (A1) implies

$$\hat{W}_Y + x\left(\frac{\theta_O - \theta_Y}{x+\theta_O}\right)\hat{H}_Y = \hat{W}_O + \frac{x}{x+\theta_O}\hat{C}_Y$$

from which it follows that

$$\begin{aligned} Var(\hat{W}_Y) - Var(\hat{W}_O) = &\left(\frac{x}{x+\theta_O}\right)^2 Var(\hat{C}_Y) + 2\frac{x}{(x+\theta_O)}Cov(\hat{W}_O, \hat{C}_Y) \\ &- \left[x\left(\frac{\theta_O - \theta_Y}{x+\theta_O}\right)\right]^2 Var(\hat{H}_Y) - 2x\left(\frac{\theta_O - \theta_Y}{x+\theta_O}\right)Cov(\hat{W}_Y, \hat{H}_Y) \end{aligned} \tag{OA3}$$

Note then that the second row in (OA3) is negative. However, the presence of the terms $\left(\frac{x}{x+\theta_O}\right)^2 Var(\hat{C}_Y)$ and $2\frac{x}{x+\theta_O}Cov(\hat{W}_O, \hat{C}_Y)$ imply that for this model economy we cannot rule out its ability to match the twin facts.

To gain intuition for why this model could at least qualitatively account for our empirical labor facts it is first useful to note that in the case where the preferences for the young (old) are also as in GHH (KPR) then this model economy corresponds to proposition 3. As such, the model cannot jointly match the labor market facts. Specifically, note that in that case equation (OA2) implies that the necessary condition to match the hours fact continues to be $\theta_O > \theta_Y$ but then equation (OA3) implies that

$$Var(\hat{W}_Y) - Var(\hat{W}_O) = -\left[x\frac{\theta_O - \theta_Y}{x+\theta_O}\right]^2 Var(\hat{H}_Y) - 2x\left(\frac{\theta_O - \theta_Y}{x+\theta_O}\right)Cov(\hat{W}_Y, \hat{H}_Y) < 0$$

and the model cannot match the wage fact.

To summarize, this example shows: If two groups differ in their labor supply wealth effect, then as long as the group with a stronger wealth effect also has a weaker substitution effect it may be possible for the twin facts to be matched.

Intuition

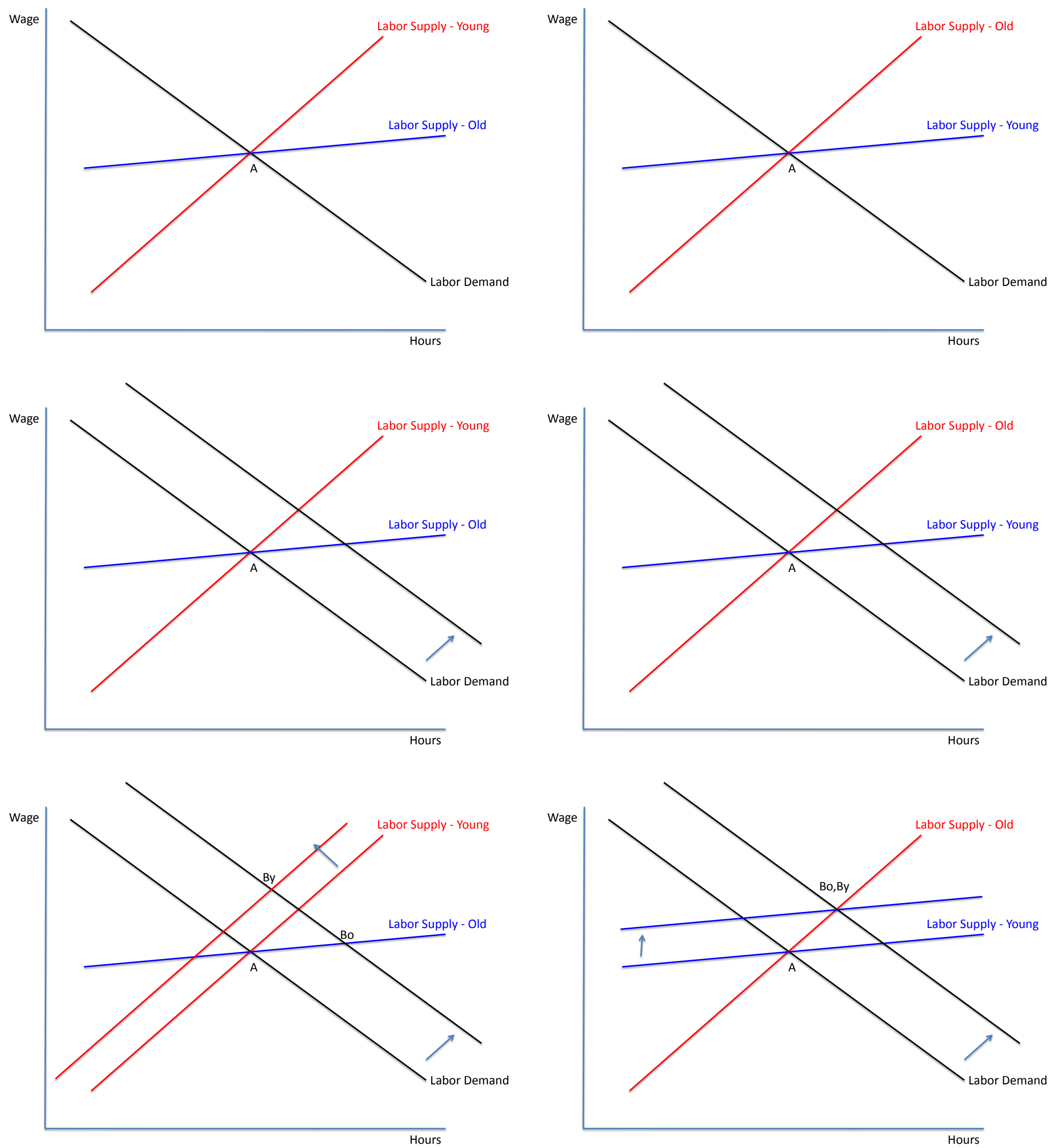
An analytical solution of the variances of age specific hours and wages is not possible since it requires a characterization the dynamic evolution of all variables. Thus, in what follows we focus on the response of variables at the impact period of the shock. Importantly, recall that Proposition 1 established that at the impact period, irrespective of the labor supply differences, it is impossible for the response of young hours and young wages to be greater than for the old. However, via simple graphical analysis we illustrate below that with counteracting wealth and substitution effects the labor market responses are not strictly contradicted.

Specifically, assume a model economy where the old have no wealth effect (i.e. have GHH preferences) while the young do have a wealth effect. The left column of panels in Figure OA1 shows the situation where the young *also* have a stronger substitution effect (i.e. the necessary condition from the example above *does not hold*), while the right column shows the situation where the young have a weaker substitution effect (i.e. the necessary condition holds). In either case, the top row displays the model in initial steady state. The key difference between the two situations is that the young labor supply curve is flatter when the necessary condition holds.

Consider a labor demand shock. In the left column, the steepness of the young labor supply curve means that the substitution effect makes young hours less responsive than old hours, and young wages more responsive than old wages (the middle-left panel). Adding the young's wealth effect (the bottom-left panel) only exacerbates the situation because the wealth response increases the wage responsiveness and decreases the hours responsiveness. Since in this situation the substitution effect

Figure OA1: Intuition for Necessary Condition on Wealth/Substitution Effects

Condition does not hold **Condition holds**



made young hours less responsive than old hours, the wealth effect mitigates the model's ability to match the hours volatility fact.

On the other hand, in the right column the young labor supply curve is flatter. Now the substitution effect implies that young hours response is greater than the old hours response (the middle-right panel) while the young wage response is less than the old wage response. Adding the wealth effect increases young wage responsiveness, counteracting the substitution effect's force (the bottom-right panel). The price paid for increasing the wage response is a decreased hours response, but it can be the case that this decrease is outpaced by the substitution effect's large hours responsiveness.

In summary, this example shows that when the wealth effect and substitution effects countervail one another then the the labor market responses are not strictly contradicted. We now turn to numerical results which support this intuition.

Numerical Results

In what follows we extend the analysis by considering more general functional forms for the young and old preferences, which necessitate numerical analysis. The presentation of the household's maximization problem is identical to that of the paper.

Given the representative household framework, wealth evolves according to the same capital stock, K_t, for both young and old workers. While this appears restrictive at first, it is not so because of our specification for preferences.

Specifically, we adopt the utility representation studied in Jaimovich and Rebelo (2009). The advantage of these preferences is that they allow for an arbitrarily large range of wealth and substitution effects. The utility functions are given by:

$$\begin{aligned} U_Y(C_{Yt}, N_{Yt}) &= \log\left(C_{Yt} - \psi_Y N_{Yt}^{1+\theta_Y} X_{Yt}\right), \quad X_{Yt} = C_{Yt}^{\gamma_Y} X_{Yt-1}^{(1-\gamma_Y)}, \\ U_O(C_{Ot}, N_{Ot}) &= \log\left(C_{Ot} - \psi_O N_{Ot}^{1+\theta_O} X_{Ot}\right), \quad X_{Ot} = C_{Ot}^{\gamma_O} X_{Ot-1}^{(1-\gamma_O)}. \end{aligned}$$

As Jaimovich and Rebelo (2009) show, γ_i controls the strength of the wealth effect on labor supply, for $i = Y, O$. Thus, recalling Figure OA1, the parameter θ_i governs the slope of the labor supply curve, while γ_i governs the wealth effect's shift of the labor supply curve. Our analysis allows for arbitrarily large differences in these key properties of the labor supply curve between young and old.

Therefore, the necessary condition described above boils down to a sign restriction: $\theta_Y > \theta_O$ and $\gamma_Y < \gamma_O$, or $\theta_Y < \theta_O$ and $\gamma_Y > \gamma_O$. Our simulations will show whether, even in very general preference specifications, this sign restriction remains necessary for the model to match the twin facts.

The production function To conduct the numerical analysis we need to specify a production function that uses capital, K_t, young labor, H_{Yt}, and old labor, H_{Ot}, as inputs. To impose *symmetry in cyclical labor demand characteristics*, we consider the following constant elasticity of substitution production function: $Y_t = A_t K_t^{\alpha}\left[\mu H_{Yt}^q + (1-\mu) H_{Ot}^q\right]^{\frac{1-\alpha}{q}}$. [37]

Business cycles in our analysis are driven by exogenous shocks to productivity, A_t. This stochastic process is identical to that considered in the paper.

The representative firm maximizes over the choice of factor inputs, taking all prices as given. In equilibrium, households and firms are optimizing and all markets clear. In particular, $H_{Yt} = s_Y N_{Yt}$ and $H_{Ot} = (1 - s_Y) N_{Ot}$.

The simulations Here we discuss the choice of parameter values under which we evaluate the model, listed in the top panel of Table OA9.

[37] When $q = 1$, we obtain the standard RBC model which treats all labor input as homogenous.

Table OA9: Simulation Parameter Values

Parameter	Range of values considered
q	$\{-10, -1, -0.5, 0, 0.5, 1\}$
θ_Y	$\{0, 0.5, 1, 1.5, 2, 3\}$
θ_O	$\{0, 0.5, 1, 1.5, 2, 3\}$
γ_Y	$\{0, 0.05, 0.1, 0.15, \ldots 0.85, 0.9, 0.95, 1\}$
γ_O	$\{0, 0.05, 0.1, 0.15, \ldots 0.85, 0.9, 0.95, 1\}$
Parameter	**Value set**
ρ	0.95
α	0.36
β	0.985
δ	0.025
s_y	0.35

Notes: The parameter q governs the elasticity of substitution between young and old hours in production; θ_Y and θ_O govern the Frisch elasticity of labor supply; γ_Y and γ_O govern the wealth effect on labor supply. We set the following parameters to one value for every simulation: ρ the autocorrelation of technology shocks, α the capital share of production, β the time discount, δ the depreciation rate and s_y the share of young in the household. See the text for detailed explanations.

The elasticity of substitution between young and old hours in production is given by $\frac{1}{1-q}$. At the upper end of the range, $q = 1$ corresponds to the case of perfect substitutes, while $q = 0$ corresponds to the case of unit elasticity of substitution (the Cobb-Douglas case) between young and old hours. We also explore lower values of q for which H_{Yt} and H_{Ot} are more complementary than the Cobb-Douglas case.

The Frisch labor supply elasticity for young (old) workers is given by $\frac{1}{\theta_Y}(\frac{1}{\theta_O})$. A natural benchmark in the quantitative literature corresponds to "indivisible labor" case, where $\theta_Y = \theta_O = 0$. [38] However, several recent papers investigating the relationship between individual- and aggregate-level elasticities suggest values between 0.7 and 3.[39] We thus consider values of θ_Y and θ_O that cover this range.

Finally, the strength of the wealth effect on labor supply is given by γ_Y and γ_O. We consider parameterizations that span the entire range between the admissible values of γ in [0,1]. Given that the literature provides little guidance on the appropriate value of γ, we use increments of 0.05, yielding 21 values of the wealth effect parameter for the young, and 21 values for the old.

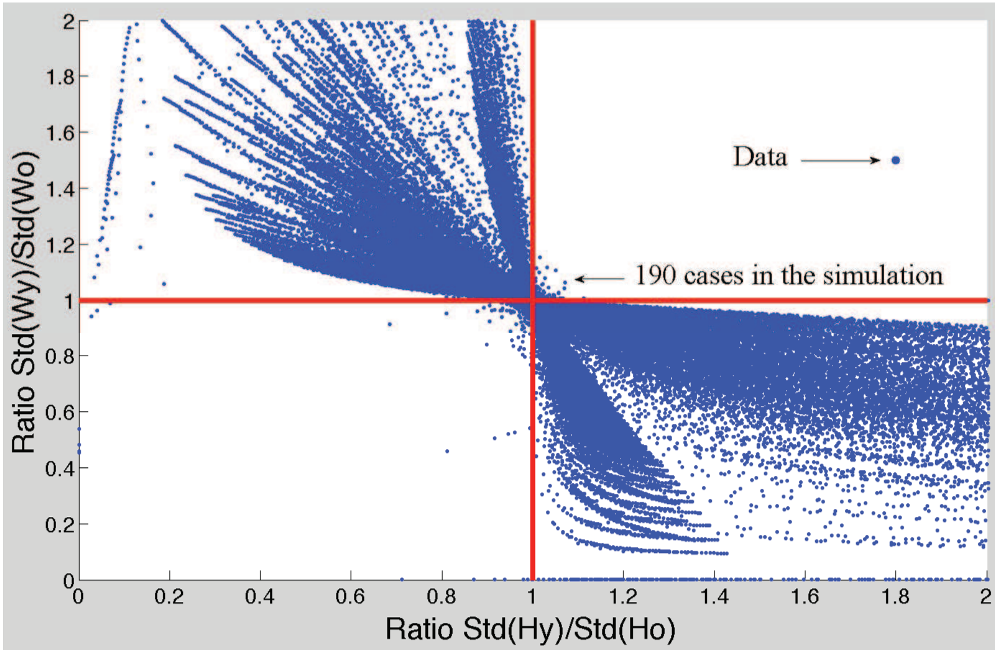
Overall, we investigate six values of q, five values of θ_Y and θ_O, and 21 values of γ_Y and γ_O. We then simulate the model $66,150$ times where each solution corresponds to a different combination of parameters. The remaining parameters of the model are set at standard values, listed in in the bottom panel of Table OA9.

Out of all 66,150 cases, we find 190 cases where we are able to *qualitatively* reproduce the labor market facts, and indeed in all of these cases we find that the sign restriction discussed above is

[38] See Rogerson (1988) and Hansen (1985).

[39] See for example Rogerson and Johanna Wallenius (2009) and Andres Erosa, Luisa Fuster and Gueorgui Kambourov (2010) and references therein.

Figure OA2: Scatter Plot of Relative Volatilities in Simulations and the Data



Notes: The twin facts are that young hours and young wages are more volatile than old hours and old wages, respectively. The value obtained from the March CPS, 1964-2010, is marked. It lies in the northeast quadrant. Also plotted are the simulated values from the 66,150 parameterizations of the labor supply model.

satisfied. Whenever the wealth effect is stronger for the young than for the old ($\gamma_Y > \gamma_O$) then the substitution effect is stronger for the old then for the young ($\theta_Y \leq \theta_O$). Vice-versa, whenever the wealth effect is stronger for the old than for the young ($\gamma_Y < \gamma_O$) then the substitution effect is stronger for the young then for the old ($\theta_O \leq \theta_Y$). That is, as discussed above, the wealth effect and substitution effect must go in "opposite" ways in order for labor supply differences alone to even *qualitatively* deliver the twin facts.

How close are the relative volatilities, $\frac{Std(H_Y)}{Std(H_O)}$ and $\frac{Std(W_Y)}{Std(W_O)}$ to the values we estimate in the data? Considering the hours volatility fact, the median, mean, and maximum value of $\frac{Std(H_Y)}{Std(H_O)}$ in the 190 cases is 1.01, 1.02, and 1.10, respectively. Meanwhile the value in the data is 1.85. Considering the wage volatility fact, the median, mean, and maximum value of $\frac{Std(W_Y)}{Std(W_O)}$ is 1.01, 1.02, and 1.15, respectively. The value in the data is 1.50. In other words, these cases are *quantitatively* unable to match the values estimated in the US data.

Figure OA2 makes the point clear. On the y-axis we plot the ratio of the volatilities of young wages to old wages $\frac{Std(W_Y)}{Std(W_O)}$, while on the x-axis we plot the ratio of the volatilities of young hours to old hours $\frac{Std(H_Y)}{Std(H_O)}$. Each dot in the scatterplot is the relative volatility of young-to-old hours and wages from one of the 66,150 simulations.[40] The northeast quadrant (where $\frac{Std(H_Y)}{Std(H_O)}$ and $\frac{Std(W_Y)}{Std(W_O)}$ are above one) is the one where the data lies.

The vast majority of cases lie in either the northwest or southeast quadrants where one of the ratio of the volatilities is less than 1. This reinforces our analytical findings that generating greater

[40]In some of the 66,150 cases, either $Std(H_O)$ or $Std(W_O)$ are very close to zero, generating very large ratios. These are excluded in the figure.

volatility of young hours results in a lower volatility of young wages (relative to the old), and vice-versa. Moreover, the cases that lie in the northeast quadrant are concentrated far from the relative volatilities observed in the data – differences in labor supply cannot *quantitatively* deliver the twin facts.

We conclude from this analysis that the necessary condition holds quite generally. Thus, differences in labor supply qualitatively deliver the twin facts by diametrically opposing the young's wealth and substitution effects relative to the old. That is, if the young have a stronger wealth effect they must also have a weaker substitution effect, or vice-versa. But then this tension leads to the fact that the model increases $\frac{Std(H_Y)}{Std(H_O)}$ by decreasing $\frac{Std(W_Y)}{Std(W_O)}$, or vice-versa, which hinders the model from generating the two volatility ratios near their values in the data. In summary, differences in labor supply seem to be an unpromising channel, at least on their own, for explaining the twin facts as estimated in U.S. data.

C Estimation

We use three lagged birth rates as our instruments: The first is the birth rate 22 years prior, which represents the middle point of the young age group. The other two are birth rates 35 and 40 years earlier, which represent the old age group. Other choices of lagged birth rates lead to similar results. The results using these three instruments for both equations (11) and (12) are reported in Panel A of Table OA10. One can see that the material difference in these estimates vis-a-vis Table 4 is the strength of the instruments for equation (12). This is a sensible result. The regressor identifying ρ is the growth rate of old hours per unit of capital, $\Delta \log(H_{Ot}/K_t)$. This variable is naturally related to fluctuations in the supply of old hours which require adjustment of the capital stock. The birth rate lagged 35 years effectively captures the influx of "new" old hours (coming from "new" old workers transitioning the young group). The 22- and 40-year lagged birth rates are naturally less connected to this variable: The first captures fluctuations in young hours (not old) while the second captures fluctuations in workers who have supplied old hours for some time (and for whom, therefore, the capital stock has already been adjusted). This is exactly the empirical result we find. The 35-year lagged birth rate is significantly correlated with the growth rate of old hours per unit of capital, while the other two instruments are not. Meanwhile, since (11)'s endogenous regressor is a combination of growth rates in young hours and output, we reason that all three birth rates have explanatory power. Thus our main results in Table 4 use the three lagged birth rates as instruments in (11) but only the 35-year lagged birth rate as an instrument for (12).

If instead we restrict either equation to be estimated by only one instrument, we obtain the Panel B of Table OA10. Here, since the endogenous regressor identifying σ involves young hours, we use the 22-year lagged birth rate. This is significantly correlated with $\Delta \log H_{Yt} - \Delta \log Y_t$ as column four reports. There are no over-identifying restrictions in this case so that the J-statistic is undefined. The resulting standard errors are much larger which leads to the F-test of $\sigma = \rho$ being borderline. We use the overidentifying restrictions (that is, use all three instruments) in our benchmark results of Table 4 because we see no *a priori* reason why we should not and because the resulting estimates are more tightly estimated. Panel B shows that the point estimate for σ is the only one materially affected by this choice, and the effect is modest.

Our benchmark results use Andrews' (1991) quadratic-spectral kernel with Newey and West's (1994) optimal bandwidth. Panel C reports results for Newey and West's (1987) Bartlett kernel instead, again with optimal bandwidth chosen according to Newey and West (1994). Panel D reports instead results for Newey and West's (1987) kernel using only one lag.[41] Panel E reports one-step (using an identity weighting matrix) and Panel F reports iterated GMM results. Using

[41]We use the Bartlett kernel here since the desire is to analyze the effect of restricting the serial correlation picked up by the HAC estimator. Since the Bartlett kernel involves truncation it is a natural candidate for this exercise; since the quadratic-spectral kernel does not truncate it less effectively captures the spirit of this sensitivity analysis. Nevertheless we report estimates from Andrews' (1991) HAC estimator with the smallest possible bandwidth yields $\hat{\sigma} = 0.615$ (0.227), $\hat{\rho} = 0.255$ (0.231), $J = 0.133$ and $\sigma = \rho$ F-test p-value 0.283.

Table OA10: Alternative Empirical Specifications

	Point Estimate	Standard Error	First Stage F-Statistic	J-test p-value	$\sigma = \rho$ p-value
			A. All Instruments		
σ	0.656	0.022	10.829	0.592	< 0.001
ρ	0.231	0.011	5.067		
			B. One Instrument per Equation		
σ	0.600	0.225	19.656	–	0.105
ρ	0.199	0.027	13.891		
			C. Newey-West Standard Errors		
σ	0.648	0.142	10.829	0.327	0.013
ρ	0.206	0.084	13.891		
			D. Limited Residual Serial Correlation		
σ	0.626	0.217	10.829	0.098	0.289
ρ	0.266	0.256	13.891		
			E. One-Step		
σ	0.650	0.192	10.829	–	0.188
ρ	0.199	0.281	13.891		
			F. Iterated		
σ	0.741	0.055	10.829	0.457	< 0.001
ρ	0.222	0.017	13.891		

Notes: Data from the March CPS, 1964-2010. J denote's Hansen's J-test; $\sigma = \rho$ denotes the F-test of the null that the two parameters are equal. Note: For several of the specifications the first stage F-statistic is indeed identical to the benchmark results reported in Table 4.

Newey and West (1987) standard errors yields little change to any of our estimates. Limiting the residual serial correlation estimated by the HAC estimator affects the point estimates mildly, the standard errors much more, but is rejected by the test of overidentification. One-step and iterated GMM point estimates are within the conventional confidence intervals of our benchmark results.

We interpret Table OA10 as evidence that the benchmark estimates presented in Table 4 are robust.

D Data

From the BEA via Haver's USECON database we obtain the following aggregate series: Real GDP per Capita is LXNFA, the GDP Defaltor is LXNFIA, Aggregate Hours is either constructed from the CPS data below[42] or taken from private aggregate hours LHTPRIVA (both yield similar results for construction of cyclical measures or calculation of data moments in our quantitative exercise), the Labor Share is LXNFBL, Capital Share is one minus the Labor Share, Real Capital per Capita is given by MFPNFKH, and Birthrates are from POPBR.

Data on age-specific hours, employment shares, and wages is constructed from the Current Population Survey (CPS) conducted by the Census Bureau. We use the surveys from 1964-2010 since the 1963 survey contained no education information. To obtain wage data, we use questions in the March CPS about income obtained in the previous (last) year. In order to turn this income data into wage data, we must know how many hours the individual worked last year. The hours for the previous year are constructed as the number of weeks worked last year multiplied by a measure of how many hours-per-week were worked by the individual last year. We modify Krusell et al.'s (2000) imputation methods the hours-per-week from the data on how many hours the individual worked *in the previous (last) week*. Our measure of hours-per-week is different than Krusell et al.'s (2000) in the following. We note whether the worker described her work last year as either full-time (FT) or part-time (PT). Her last week's hours are imputed as the hours-per-week only if the value falls within believable values, given that her work last year was either FT or PT. If her previous week's hours are not consistent with FT or PT work, we impute a "disaggregated" group average as the hours-per-week; by contrast, Krusell et al. (2000) impute a "disaggregated" group average only if the worker reported that she worked last year but worked zero hours last week.

Our "disaggregated" groups are formed by dividing respondents by age, education, gender, and last year's FT/PT status. Given that there are eleven 5-year age bins (15-19,20-24,. . . ,60-64,65+), 5 education bins (below HS, HS, some college, college graduate, postgraduate work), 2 genders, and a FT or PT status, there are 220 possible groups. Our "disaggregated" groups combine education bins for some age-gender-FT/PT groups to ensure that for every year in 1964-2010 our "disaggregated" groups each have at least fifty members.[43] This is done so that the "disaggregated" group average is not overly reliant on only a few observations.

Conditional on the other characteristics we consider, we use the information on PT and FT as follows:

- If a person claims to be PT last year and works between 1 and 34 hours last week, we impute their hours-per-week last year as their hours last week; otherwise (they worked 0 or more than 34 hours last week) they are imputed the group average
- If a person claims to be FT last year and works 35 or more hours last week, we impute their hours-per-week last year as their hours last week; otherwise (they worked 0-34 hours last week) they are imputed the group average

Let g' be the part of g with hours last week that are FT-status-fitting for imputation purposes (given the FT/PT nature of g), and g'' be those whose hours last week are not FT-status-fitting. Let h_i, m_i, y_i, and μ_i be worker i's hours last week, number of weeks worked last year, wage and salary income last year, and CPS Person weight, respectively.[44] Then the measures of group g's

[42] When constructing aggregate hours from the CPS, we use the 1962 and 1963 surveys since we do not need to form disaggregated groups based on educational attainment. We need these earliest surveys in order to provide the lag of aggregate hours onto which we project when creating cyclical measures. Forming such groups for the purposes of imputing missing values in last week hours has only minimal effects which are negligible when we do so for 1964-2010 when education data is provided.

[43] Additionally cutting by race (white/nonwhite) does not change matters much.

[44] In the March supplement, we have both a CPS Basic Person weight, and a CPS Supplemental Person weight. Personnel at the Census Bureau have advised us to use the latter for all the data questions we are addressing, even though some of these data are not part of the March Annual Supplement.

"disaggregated" group average, weight, hours worked last year, and income last year are

$$
\begin{aligned}
h_{g'} &= \frac{1}{\sum_{i \in g'} \mu_i} \left(\sum_{i \in g'} h_i \mu_i \right) \\
\mu_g &= \sum_{k \in g} \mu_k \\
h_g &= \frac{1}{\mu_g} \left(\sum_{i \in g'} h_i \mu_i + \sum_{j \in g''} h_{g'} \mu_j \right) \\
y_g &= \frac{1}{\mu_g} \left(\sum_{k \in g} y_k \mu_k \right)
\end{aligned}
$$

Let γ be a set of gs: this is a larger group, such as all workers in the 15-19 age category, comprised of smaller "disaggregated" groups. Our construction of an efficiency wage measure for γ is similar to that of Krusell et al. (2000): our efficiency measurement f for each g is the average of their wage (y_g/h_g) for the years 1985-1989.[45]

$$
W_\gamma = \frac{\sum_{g \in \gamma} y_g \mu_g}{\sum_{g \in \gamma} h_g f_g \mu_g}
$$

It is worth mentioning that the March CPS has a specific question "On average, how many hours per week did you work last year, when you worked?" starting in 1976. We find that making sure the hours imputation is FT-status-fitting leads to hours measures that are close to the post-1976 question when both are available. By ignoring the FT-status, one underreports the groups' hours.

Our data on hours come directly from the hours last week question. Likewise, our labor force share data comes from a labor force status question pertaining to last week. Our data on wages come from the wage and salary income last year question, screening for self-employed and farm-working individuals. To link our labor income data with hours worked data, we use the hours worked last year in the young/old hours data used for GMM estimation of σ and ρ.

[45] Krusell et al. (2000) use the wage in 1980 as the efficiency measurement. We use an average of the wage to allow for the possibility that the efficiency measure varies over the cycle. Hence, by averaging over five years we aim to smooth the efficiency measurement. The results remain the same using either efficiency measurement.