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Note

Income effects and indeterminacy in a calibrated one-sector growth model

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Abstract

This note analyzes how the indeterminacy of competitive equilibrium in one-sector growth models depends on the magnitude of the households' income effect on the demand for leisure. Since I am interested in quantitatively characterizing regions of indeterminacy, I use the Jaimovich and Rebelo [N. Jaimovich, S. Rebelo, Can news about the future drive the business cycle? Mimeo, Northwestern University, 2007] preferences that span a wide range of income effect values. I find that indeterminacy can occur for levels of aggregate-returns-to-scale that are well within recent empirical estimates. For these regions of indeterminacy, the model, when driven solely by sunspot shocks, generates second-moment properties that are consistent with the U.S. data at the business cycle frequency.

JEL classification: E10; E32; D90

Keywords: Indeterminate equilibria; Utility function; Sunspot shocks; Business cycles

1. Introduction

The goal of this note is to investigate how the existence of equilibrium indeterminacy depends on the magnitude of income effect on the demand for leisure.[1] Specifically, I study a class of one-sector infinite-horizon models of capital accumulation and external effects in production with bounded returns in which the representative agent values consumption and leisure. Within

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[1] I refer to an equilibrium indeterminacy as an equilibrium in which multiple paths converge toward the same steady state.

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the literature that has addressed the issue of indeterminacy in this class of models, Benhabib and Farmer [4] is the most relevant reference.[2] Benhabib and Farmer [4] analyze a Cobb–Douglas economy with endogenous labor supply and they prove the existence of equilibrium indeterminacy. The specific utility function that they analyze is characterized by the presence of a positive income effect on the demand for leisure.

Early criticism of the Benhabib and Farmer [4] model questioned the empirical plausibility of its intermediacy result because it required a level of aggregate returns to scale (ARTS) in the production function that was at odds with the existing estimates. Subsequent work within this area has resulted in examples of model economies that are characterized by indeterminacy with much lower and more empirically plausible levels of ARTS.[3]

While these subsequent studies analyzed different economic environments, one common feature they shared was the assumption of a utility function characterized by the presence of positive income effect on the demand for leisure. In this note, I shift the focus of attention from the dependency of indeterminacy on the degree of ARTS to the dependency of indeterminacy on the magnitude of income effect on the demand for leisure. The first result that emerges from this analysis is as follows: when the utility function belongs to the class of functions that exhibit no income effect on the demand for leisure, an indeterminate equilibrium cannot exist. That is, in models with endogenous labor supply, the presence of income effect on the demand for leisure is a necessary condition for the existence of indeterminacy. Given the prevalence of utility functions that exhibit no income effect in the macroeconomic literature, this result is of particular interest.[4]

However, the quantitative implications of this necessary condition are somewhat limited. For example, it is not informative in terms of the minimum degree of income effect required for indeterminacy to exist. Similarly, this result does not allow for characterization of regions of uniqueness and regions of indeterminacy of equilibria as a function of varying degrees of income effect. Moreover, a trade-off is to be expected: an income effect that is "too strong" would tend to reduce the plausibility of equilibrium indeterminacy as this would, all other things equal, reduce the labor supply of agents in response to a shock that induces a rise in the marginal utility of wealth and impinge on the plausibility of indeterminacy. From this quantitative point of view, these shortcomings are driven by the fact that the two classes of utility functions most widely used in the macro literature—the class of preferences discussed in King, Plosser, and Rebelo [19] (KPR) and the Greenwood, Hercowitz, and Huffman [12] (1988) utility function—represent

[2] Kehoe et al. [16], Kehoe et al. [17], and Spear [27] among others have grappled with the issue of indeterminacy in the one-sector growth model with bounded returns. The differences among these papers lie in the type of the external effect considered.

[3] See, for example, Aiyagari [1], Basu and Fernald [3], Benhabib and Farmer [5,6], Benhabib and Nishimura [7], Wen [31], Bennet and Farmer [8], Harrison [13], Harrison and Weder [14], Matsuyama [20], Romer [23], Schmitt-Grohé [24,25]. See also Shell [26] and Benhabib and Farmer [6] for two excellent surveys of models with sunspot equilibria.

[4] A survey of the extensive use of this utility function is beyond the scope of this note. In general, the literature has emphasized that the "no income effect" utility functions improve the ability of various models to reproduce some business cycle facts. For some prominent examples, see Greenwood et al. [12] who analyze business cycle dynamics using a momentary utility function exhibiting no income effect. Mendoza [21] introduces a generalized version of the utility function used in Greenwood et al. [12] in order to examine business cycle dynamics in a small open economy. Correia et al. [10] analyze the business cycle dynamics in a small open economy with respect to two different utility functions: one similar to that of Greenwood et al. [12] and one belonging to the class described by King et al. [19]. Perri and Neumeyer [22] use the utility function in Greenwood et al. [12] to analyze the importance of movements in the interest rate in explaining business cycle dynamics in small open economies.

two polar cases of constant income effect. As such, they do not enable one to conduct this type of analysis. Therefore, in order to characterize the dependence of indeterminacy on different degrees of income effect, it is necessary first to introduce a utility function flexible enough to encompass varying degrees of income effect.

The preferences introduced in Jaimovich and Rebelo [15] (JR) turn out to be suitable candidates for this task. The JR preferences nest as special cases the KPR and GHH utility functions. Moreover, they are flexible enough that they can easily span the entire range of income effect that lies between these two utility functions. Introducing the JR utility function into the analysis allows me to quantitatively characterize regions of indeterminacy as a function of varying degrees of income effect and ARTS. Based on this characterization, I find a lower and an upper bound for the magnitude of income effect consistent with indeterminacy. By allowing for varying degrees of income effect, I find that indeterminacy occurs for levels of ARTS that are well within the recent empirical estimates. Finally, for these regions of indeterminacy, I simulate the model driven solely by sunspot shocks. I find that the second-moment properties of this model are generally consistent with the U.S. data at the business cycle frequency. These results suggest that the one-sector growth model can exhibit indeterminacy and generate second moment that are consistent with those observed in the U.S. data for plausible parameter values once varying degrees of income effect are introduced into the analysis

It is important to emphasize that similarly to the vast majority of the literature that studies the nature of the sunspots equilibria arising under production externalities as those studied in Benhabib and Farmer [4], I study local sunspots; i.e. I study sunspots in the vicinity of the steady state. Hence the sunspots are not global (see Wang and Wen [28,29] for excellent discussions of global sunspots in models of imperfect competition). The rest of the note is organized as follows. I begin with example in which the connection between the indeterminacy of an equilibrium and the presence of income effect on the demand for leisure is established. The note then continues by introducing the Jaimovich–Rebelo preferences and shows how they are able to encompass varying degrees of income effect. I then continue by analyzing the interaction between these varying degrees of income effect and the existence of indeterminacy. The note concludes by evaluating the second moment properties of the model for different degrees of income effect and ARTS. Section 3 concludes.

2. Varying degrees of income effect and indeterminacy

I begin by studying an example that highlights the necessity of the income effect on the demand for leisure for the occurrence of indeterminacy. The model economy is almost identical to the model economy in Benhabib and Farmer [4]. The only difference with respect to the environment analyzed there is the assumption that the utility function is the one introduced by Greenwood et al. [12]. This utility function is characterized by the presence of no income effect on the demand for leisure.

At each point in time, the representative agent maximizes his utility from streams of consumption and leisure according to

$$\max_{C_t, H_t} \int_0^\infty \log\left(C_t - \psi H_t^{1+\chi}\right) e^{-\rho t}\, dt$$

subject to the law of motion of capital

$$\dot{K}_t = (r_t - \delta) K_t + W_t H_t - C_t + \Pi_t \tag{2.1}$$

where C_t, K_t, and H_t denote, respectively, consumption, capital holdings, and hours worked at period t. The time endowment is normalized to one, ρ denotes the discount rate, and δ is the depreciation rate. The households own the capital stock and take the equilibrium rental rate, r_t, and the equilibrium wage, W_t, as given. Finally, the households own the firms and receive any profits, Π_t. Similarly to the Benhabib–Farmer model, the production function is given by

$$x_t = K_t^q H_t^b \left[\overline{K}_t^{q\theta_1} \overline{H}_t^{b\theta_2}\right]$$

where K_t and H_t denote the capital and hours worked, used by the firm, at period t. $\overline{K}_t$ and $\overline{H}_t$ denote the aggregate capital and aggregate hours worked at period t, respectively. Moreover, as in Benhabib and Farmer [4] I assume

$$q = 1 - b,$$
$$\alpha = q(1 + \theta_1) > q,$$
$$\beta = b(1 + \theta_2) > b.$$

Hence, in a symmetric equilibrium, $K_t = \overline{K}_t$ and $H_t = \overline{H}_t$, implying that the aggregate output is given by,

$$Y_t = K_t^\alpha H_t^\beta.$$

In this specific example one can show that the sign of the determinant of the dynamic system is defined by the sign of

$$\frac{\beta - (1-\alpha)(1-\chi)}{(1+\chi-\beta)} \tag{2.2}$$

which implies that the determinant is positive (a necessary condition for indeterminacy) if and only if

$$\beta \in \big((1-\alpha)(1+\chi), (1+\chi)\big). \tag{2.3}$$

Similarly, given the admissible values for β that (2.3) identifies, the trace is negative only if the following condition holds

$$\left(\frac{\delta+\rho}{q}\right)\left(\frac{\beta-(1-\alpha)(1-x)}{(1+\chi-\beta)}\right) < 0.$$

Given the admissible values for β that (2.3) identifies, this last expression is positive. Thus the trace cannot be negative whenever the determinant is positive, implying that indeterminacy cannot exist in this model example.

The result that the presence of income effect on the demand for leisure is a necessary condition for equilibrium indeterminacy holds in a general class of models.[5] Given the prevalence in the macroeconomic literature of utility functions that exhibit no income effect, this result is of particular interest. Moreover, it hints at the potential return to being able to analyze the interaction between equilibrium indeterminacy and income effect for varying degrees of the latter, beyond the limiting case of no income effect. However, as discussed in the introduction, the main limitation for conducting such a quantitative analysis is that the two classes of utility functions most widely used in the macro literature—the class of preferences discussed KPR and the GHH utility

[5] See the discussion in an earlier version of the note which can be found at http://www.stanford.edu/~njaimo/papers/income_effects_indeterminancy_110207.pdf.

function—represent two polar cases of constant income effect. Therefore, this type of analysis cannot be conducted. So, in order to be able to characterize the dependence of indeterminacy on different degrees of income effect, it is necessary first to introduce a utility function flexible enough to encompass varying degrees of income effect. As mentioned earlier, the JR preferences nest as special cases the KPR and GHH utility functions, and they span the entire range of income effect that lie between these two.

Specifically, the agents in the JR economy maximize a life-time utility U, which is defined over sequences of consumption and hours worked according to the following functional form[6]

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \psi X_t H_t^{1+\chi})^{1-\sigma} - 1}{1-\sigma} \tag{2.4}$$

where

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}. \tag{2.5}$$

The parameter $\psi > 0$ is used for accounting for the steady state of H_t. $\sigma > 0$ controls the curvature of the utility function. For the rest of the note I concentrate at the case of $\sigma = 1$.

When $\gamma = 1$, these preferences belong to the KPR class of preferences. When $\gamma = 0$, the GHH preferences are obtained. Also, the JR preferences are consistent with balanced growth in the presence of labor-augmenting or investment-specific technical progress, and they exhibit weak income effect on leisure in the short run for low values of γ. For the purposes of this note, these preferences allow me to control the magnitude of income effect by varying γ for values between the two extremes (i.e., $\gamma \in [0, 1]$). Thus I can characterize regions of indeterminacy as a function of income effect and ARTS.[7]

2.1. Dynamic Hicksian decomposition of income effect

Before performing this analysis, I discuss some general features of this utility function that will be useful for the analysis of the interaction between income effect and indeterminacy. As in Jaimovich and Rebelo [15], I follow here the approach in King [18], which discusses a dynamic version of the Hicks decomposition. This experiment is carried out as follows. First, I study the response of hours of an agent who faces a permanent increase of 1% in *TFP*.[8] For each value of $\gamma \in [0, 1]$, this permanent shock raises the lifetime utility from U^* to $U^*(\gamma)$.[9] Fig. 1, Panel *A*, graphs the response of hours worked to the permanent *TFP* increase as a function of 10 different values of γ. The strongest response of hours worked occurs with GHH preferences ($\gamma = 0$). In this case, because of the lack of income effect, hours worked are not stationary—they rise permanently in response to the permanent increase in the real wage rate. With KPR preferences ($\gamma = 1$), hours worked converge back to the steady state after the shock, but the

[6] As I am interested in eventually simulating the model (see below) and comparing its predictions to the U.S. data, I shift to a discrete-time model.

[7] In the case of $\gamma = 1$ the preference does not nest the "indivisible" preferences. In this case when $\chi = 0$ one can show that the Frisch labor supply elasticity is given by $\frac{1-\psi H}{\psi H}$.

[8] To isolate the effects of variations in γ, I first study this decomposition for the case of an aggregate production function that exhibits constant returns to scale. See below for a similar decomposition for different levels of ARTS.

[9] In order to have the steady-state allocations not depend on γ, and thus U^* remain the same prior to the *TFP* shock, ψ has to be a function of γ, $\psi(\gamma) = \frac{1}{(1+\chi)(H^*)^{\chi} \frac{C^*}{W^*} + \frac{\gamma (H^*)^{1+\chi}}{1-\rho(1-\gamma)}}$.

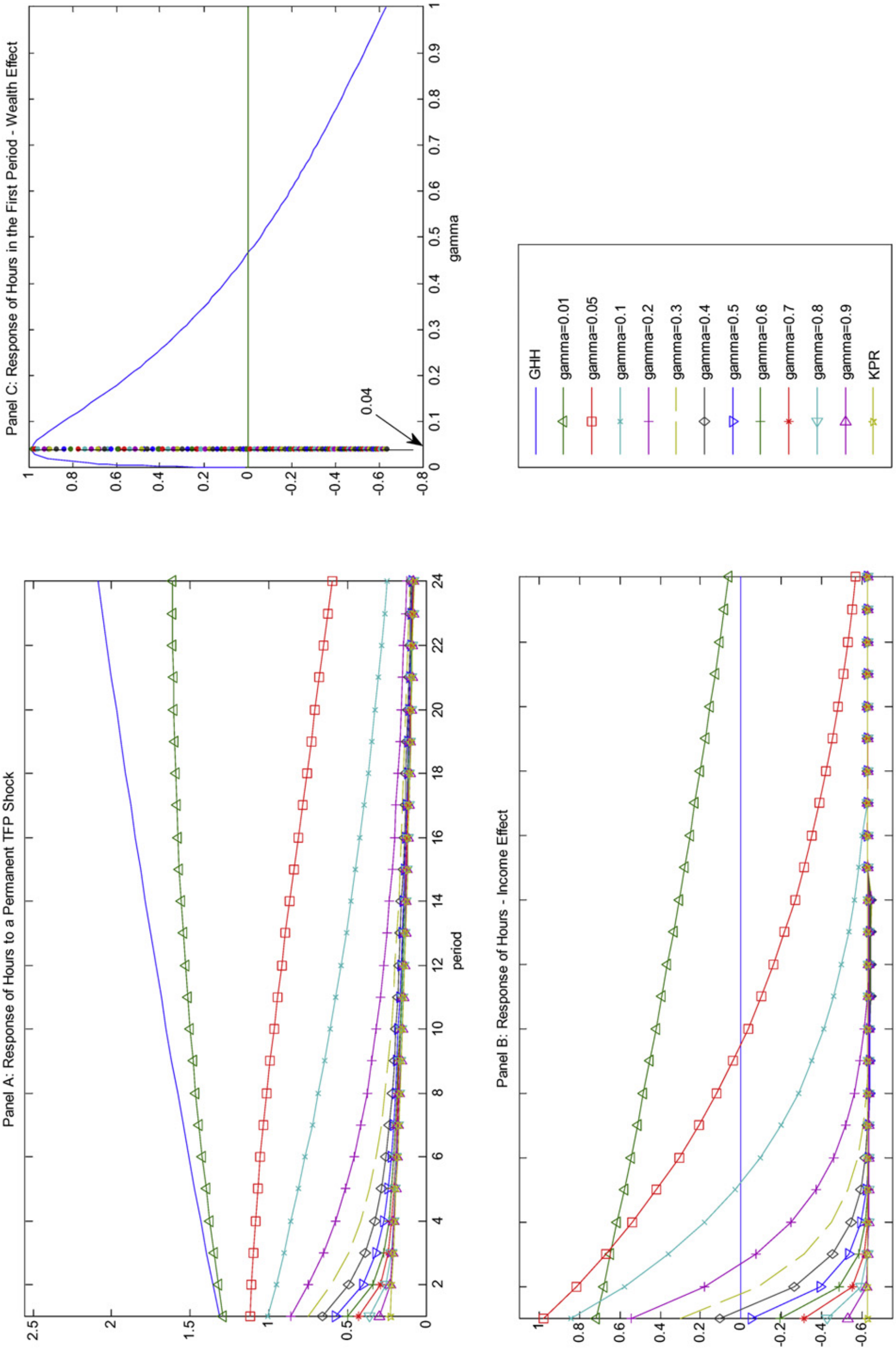
Panel A: Response of Hours to a Permanent TFP Shock
period
Panel B: Response of Hours - Income Effect
period
Panel C: Response of Hours in the First Period - Wealth Effect
gamma
0.04
GHH
gamma=0.01
gamma=0.05
gamma=0.1
gamma=0.2
gamma=0.3
gamma=0.4
gamma=0.5
gamma=0.6
gamma=0.7
gamma=0.8
gamma=0.9
KPR

Fig. 1.

short-run response of hours worked is weak. The other lines represent the response of hours worked when $\gamma \in (0, 1)$. With these preferences, hours worked also converge to the steady state, but the short-run impact of the *TFP* shock falls between those of GHH and KPR preferences.[10] Lower (higher) values of γ produce short-run responses that are closer to those obtained with GHH (KPR) preferences.

To calculate the income effect, I compute, for each $\gamma \in [0, 1]$, the path of labor supply of a household that receives an output transfer and faces wages and real interest rates that are constant at their steady state levels. The level of the transfer is computed such that the agent's utility is $U^*(\gamma)$ (without the transfer the agent's utility would be U^*). Thus, the agent solves the following Lagrangian

$$\max \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \psi X_t H_t^{1+\chi})^{1-\sigma} - 1}{1-\sigma} + \sum_{t=0}^{\infty} \beta^t \lambda_t \big(W_t H_t + (1 + r_t) A_t + \zeta_t - C_t - A_{t+1}\big) + \sum_{t=0}^{\infty} \beta^t \phi_t \big(X_t - C_t^{\gamma} X_{t-1}^{1-\gamma}\big)$$

where ζ_t is the output transfer and A_t denotes wealth at period t.

Based on this dynamic Hicksian decomposition, Fig. 1, Panel B, graphs the response of hours that are attributable to the income effect for the same 10 values of γ shown in Panel A. For the GHH preferences, the income effect is zero, whereas for the KPR preferences the income effect is negative (in both cases the income effect is constant over time). With the JR preferences, the income effect is time varying. Like the total response of hours, for $0 < \gamma \leqslant 1$, the income effect converges to that of the KPR preferences in the long run.

Panel B reveals two interesting results. First, in the short-run, there is a hump-shape in the income effect as a function of γ. Second, for certain values of γ, the short-run income effect is actually negative (i.e., inducing a short-run increase in hours worked). These two effects are easier to notice in Fig. 1, Panel C which graphs, for each different level of γ, the income effect at the period of the transfer shock. Again, the hump-shaped income effect is evident from this figure. The rationale for the negative income effect is the following. Consumption grows over time in this experiment. With this type of preferences, the growth in consumption implies that the dis-utility of work is higher in the future than in the present—for relatively lower values of γ, the future consumption growth is contemporaneously weighted more heavily, inducing an increase in hours worked.

2.2. *Discussion of the implications for indeterminacy*

In order to isolate the role of income effect on the existence of indeterminacy, I use the same model as Benhabib and Farmer [4], with one difference. The utility function of the representative agent is given by the JR preferences, as specified in (2.4) and (2.5). I will refer to this model as the Benhabib–Farmer–Jaimovich–Rebelo (BFJR) model.

[10] As long as $0 < \gamma \leqslant 1$, hours worked converge back to the steady state. The slowest convergence occurs for the lowest value of $\gamma > 0$ that is considered—$\gamma = 0.01$. This takes place after 150 periods.

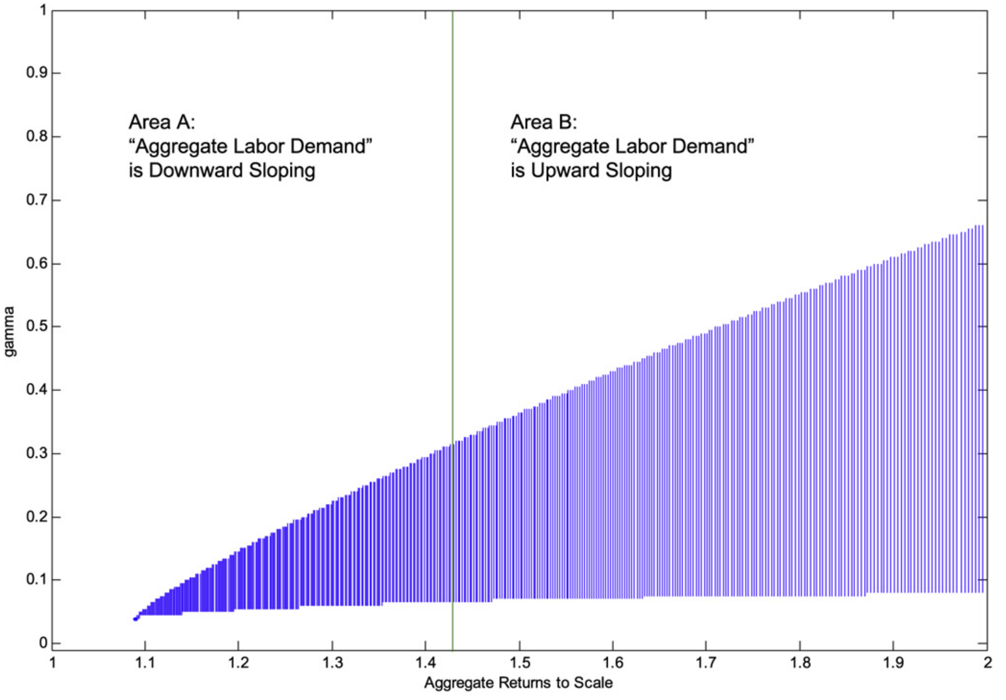

Fig. 2.

I begin the analysis by characterizing regions of indeterminacy for various levels of ARTS, $(\alpha + \beta)$, and different values of γ. Since a point estimator of γ does not exist at the literature I preform this analysis for various levels of $\gamma \in [0, 1]$. It is worth emphasizing again, that any value of $\gamma > 0$ is consistent with the stationary of hours in the U.S. data and with balanced growth in the presence of labor-augmenting or investment-specific technical progress; Thus, these empirical considerations by themselves cannot rule out any value of γ. This exercise should be thus regarded as a first step in studying the quantitative importance of income effects for indeterminacy.

The dark region in Fig. 2 emphasizes the main message of this note: the analysis of the interaction between income effect and indeterminacy yields new insights on the plausibility of the latter.[11] First of all, Fig. 2 shows that, indeed, for any level of ARTS there is no indeterminate equilibrium for the case of $\gamma = 0$ (i.e., the case of no income effect).[12] Moreover, Fig. 2 reveals that there is a minimum value of γ that is required such that an indeterminate equilibrium exists. The intuition for this result is similar in nature to the discussion in the introduction. In order for indeterminacy to exist, there has to be a strong enough reaction of the labor supply to the sunspot shock, because the labor demand curve is fixed at the period of the shock. However, as discussed in the introduction, a trade-off is to be expected: when the "aggregate labor demand" is either downward sloping (region A of Fig. 2), or upward sloping but with a smaller slope than that of the labor supply (region B of Fig. 2), then an income effect that is "too strong" would

11 In the analysis that follows, I concentrate on the case of an increase in hours worked in response to a sunspot shock.

12 This is true even for higher levels of ARTS than shown in Fig. 2.

tend to work against the possibility of indeterminacy. Indeed, Fig. 2 reveals that this is the case: for all the levels of ARTS considered there is a maximum level of γ which is consistent with indeterminacy.[13] Finally, as expected, Fig. 2 reveals that the higher the ARTS, the broader the range of $\gamma's$ for which indeterminacy exists.

Another way to glean an insight into the interaction between the income effect and indeterminacy is through analyzing the labor market equilibrium condition. Log-linearizing the first-order condition for the labor supply and consumption yields

$$\hat{h}_t = \frac{1-\psi H^{*1+\chi}}{\psi H^{*1+\chi}+\chi}\hat{w}_t + \left(\frac{(1-\gamma)\hat{c}_t + (1-\psi H^{*1+\chi})\hat{\lambda}_t}{\psi H^{*1+\chi}+\chi}\right) - \left(\frac{(1-\gamma)}{\psi H^{*1+\chi}+\chi}\right)\hat{x}_{t-1}.$$

The coefficient of $\hat{w}_t$ acts as the slope in this log-linearized labor supply condition, while the remaining expressions act as a shift variables. Assume that at period t the economy is at a steady state; thus, $\hat{x}_{t-1} = 0$. At the arrival of the sunspot shock (similar to Benhabib and Farmer [4], I model the sunspot shock as a fall in $\hat{\lambda}$), the response of hours is determined by the sign of[14]

$$\left(\frac{[1-\gamma]\hat{c}_t + [1-\psi H^{\theta}]\hat{\lambda}_t}{\psi H^{\theta} + (\theta - 1)}\right). \tag{2.6}$$

In the numerical simulations, I consider only those cases in which the labor supply is upward sloping and the "aggregate labor demand" is either downward sloping or upward sloping but with a smaller slope than that of the labor supply. Therefore, hours can become positive at the impact period only if the labor supply shifts to the "right and downward." That is, only if the sign of (2.6) is positive at the impact period of the sunspot shock can indeterminacy occur.[15] It is thus of interest to consider how variations in γ affect the value of (2.6).[16] One can show that as γ increases, the value of the coefficient on $\hat{\lambda}_t$ increases. Since the sunspot shock is modeled as a fall in the marginal utility of wealth, all other things being equal, an increase in γ reduces the plausibility of the existence of indeterminacy. Similarly, for values of γ above 0.06, the coefficient on $\hat{c}_t$ decreases as γ increases. This again implies that, all other things being equal, an increase in γ, and thus a strengthening of the income effect, reduces the plausibility of indeterminacy.

2.2.1. *Indeterminacy when "aggregate labor demand" is downward sloping*

Fig. 2 reveals that allowing for a weaker income effect than the one induced by the KPR utility function implies that indeterminacy exists even in the case of ARTS that are small enough that the "aggregate labor demand" has the "conventional" slope, i.e., negative. For example, I find that the minimum level of ARTS for which there is an indeterminate equilibrium is 1.09. This value is well within the empirical estimates found in the literature.[17] In order to better understand the occurrence of indeterminacy in the case of "downward" labor demand, I conduct a dynamic Hicksian decomposition of the hours' response to a sunspot shock in the BFJR model. That is, for each pair of ARTS and γ that gives rise to an indeterminate equilibrium in area A

[13] Fig. 1, Panel C, shows that for values of $\gamma > 0.04$, increases in γ induce a stronger income-effect on hours. Fig. 2 reveals that for all the levels of ARTS considered, the maximum level of γ is always in this region of $\gamma's$.

[14] Because of the dynamic nature of the problem, λ_t cannot be expressed solely as a function of period t variables. Rather, it is a function of the entire future sequence of variables.

[15] This somewhat "informal" argument is verified numerically.

[16] The two coefficients are independent of the degree of ARTS, which enables me to concentrate on the effects of varying γ.

[17] See, for example, Burnside et al. [9] and Basu and Fernald [2].

in Fig. 2, I introduce a sunspot shock at the first period.[18] The sunspot shock raises the lifetime utility from $U^*(ARTS)$ to $U^*(ARTS, \gamma)$. Then, for each of these pairs I follow the steps discussed in Section 2.1 and calculate the level of the output transfer such that the agent's utility equals $U^*(ARTS, \gamma)$. The two main results that emerge from this analysis are as follows. First, conditional on the level of ARTS, when indeterminacy occurs, the higher γ is, the stronger the income effect. This again reflects how increases in γ strengthen the income effect and, holding everything else constant, reduces the plausibility of indeterminacy. Second, conditional on the level of γ, the higher the level of ARTS, the weaker the income effect. This again illustrates the previous discussion of Fig. 2: the higher the ARTS, the broader the range of $\gamma's$ for which indeterminacy exists.

2.3. Second moments

In this section I analyze the business cycle properties of the BFJR model. Specifically, I consider ten different cases of ARTS that induce "aggregate labor demand" that is downward sloping: for each of these cases I find the set of $\gamma's$ that gives rise to an indeterminate equilibrium. Each of these ten sets is then discretized by increments of 0.01, resulting in 136 pairs of ARTS and γ for which the model is indeterminate. For each of these pairs, I simulate the model and compute the second moment of output, consumption, investment, and hours.[19]

For ease in presenting the simulation results for these 136 pairs, Fig. 3, Panel A, graphs the standard deviation of HP-filtered hours relative to the standard deviation of HP-filtered output for all pairs. Similarly, Panels B and C in Fig. 3 graph the standard deviation of HP-filtered consumption and HP-filtered investment relative to the standard deviation of HP-filtered output, respectively. For comparison, in the U.S. data the standard deviations of HP-filtered hours, HP-filtered investment, and HP-filtered consumption relative to the standard deviation of HP-filtered output are 0.97, 0.75, and 3.05, respectively.[20]

With respect to the volatility of hours, the model's performance is generally consistent with the data, as hours are as volatile as output.[21] Since the JR preferences are not separable across consumption and labor effort, the optimality conditions of the household problem imply that $(C_t - \psi H_t^{1+\chi} X_t)$ should be smoothed over time. For this reason, the response of labor supply to the sunspot shock induces additional movements in consumption. Indeed, as Panel B suggests, the model generates volatility of consumption that is closer to the one observed in the data than

[18] I analyze the response to a sunspot shock and not a *TFP* shock, as done previously, for two reasons. First, I am interested in learning the response of the system to a sunspot shocks because I later study the second moment properties of a model driven by this shock. Second, because the marginal utility of wealth is a jump variable, its response is not pinned down after a *TFP* shock in the case of an indeterminate equilibrium. By modeling the sunspot shock as in Benhabib and Farmer [4], the "jump" of the marginal utility of wealth at the impact period is well defined.

[19] In all the simulations I follow the approach in Benhabib and Farmer [4], and I set the volatility of the sunspot shock so that the model matches the volatility of output in the U.S. data. Also, as discussed previously, the unique case where the JR preferences are not consistent with balanced growth path is the case of $\gamma = 0$. For any other value of $\gamma > 0$ the JR preferences are such that hours are stationary. However, one concern is that in small samples the model will generate non-stationarity of hours. I thus check for non-stationarity of hours in each of the 136 simulations and find that in all the simulations the behavior of hours is consistent with balanced growth path.

[20] I use quarterly data between 1947:I–2004:IV. A smoothing parameter of 1600 is used for the filtering.

[21] It is interesting to note that, conditional on a value of γ, a higher level of ARTS will lead to a lower volatility of hours. This might be surprising, given the dynamic income effect decomposition carried out previously. However, a decomposition of the dynamic substitution effect can be conducted that shows that, conditional on a value of γ, the higher the ARTS, the lower the substitution effect, thus inducing an overall reduction in the volatility.

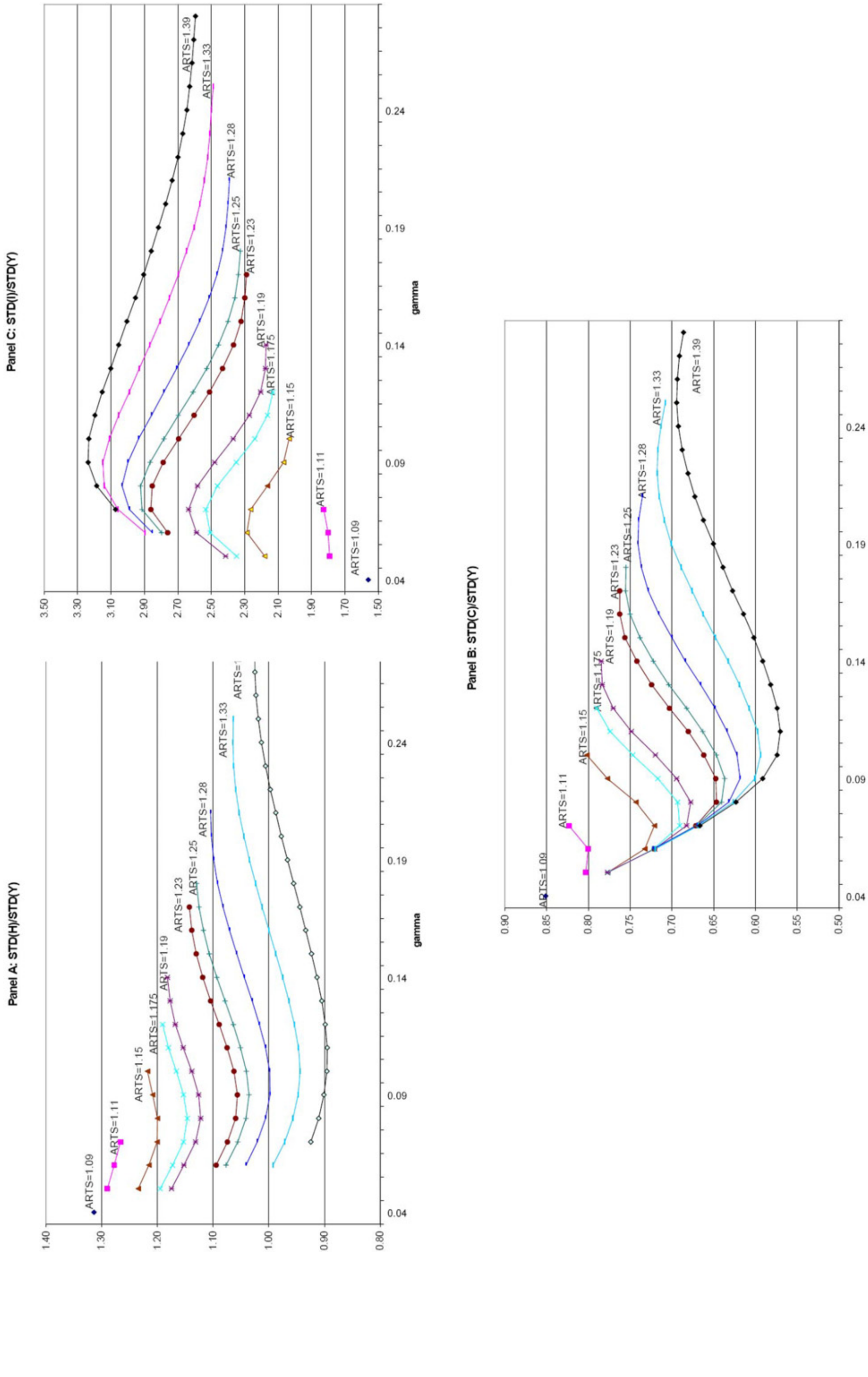
Panel C: STD(I)/STD(Y)
ARTS=1.39
ARTS=1.33
ARTS=1.28
ARTS=1.25
ARTS=1.23
ARTS=1.19
ARTS=1.175
ARTS=1.15
ARTS=1.11
ARTS=1.09
gamma
Panel A: STD(H)/STD(Y)
Panel B: STD(C)/STD(Y)

Fig. 3.

that generated by other one-sector "sunspot models," all of which assume a utility function that belongs to the KPR class (see, for example, Farmer and Guo [11] and Wen [30]. In the former the ratio equals 0.23, whereas in the latter it equals 0.04). The increased volatility of consumption in turn induces a reduction in the relative volatility of investment as compared to the results of these other models (again, for example, this equals 8.91 in Farmer and Guo [11] and 4.63 in Wen [30]) and brings it closer to the observed ratio in the U.S. data.[22]

With respect to the contemporaneous correlations of HP-filtered hours with HP-filtered output, as in the U.S. data, the model generates high correlations: all 136 correlations fall between 0.97 and 0.99 (the correlation in the U.S. data equals 0.86). Similarly, the model generates a consumption process that is highly correlated with output: the 136 correlations lie between 0.8 and 0.99 (the correlation in the U.S. data equals 0.77). Finally, with respect to investment, the model generates an investment process that is highly correlated with output as well: the correlations lie between 0.91 and 0.97 (the correlation in the U.S. data equals 0.89).

Thus, to conclude, with the sole modification of the JR preferences, the model induces (1) a variance of hours, consumption, and investment, and a contemporaneous correlation between these three variables and output that in general is consistent with the U.S. data, and (2) an empirically relevant increase (decrease) in the volatility of consumption (investment) relative to one-sector "sunspot models" that assume a utility function of the KPR class. Once again, it is worth noting that these results are obtained for values of ARTS that are close to those estimated in the U.S. data and that induce "aggregate labor demand" that is downward sloping. Moreover, these results are obtained for values of γ that induce stationary hours and are thus consistent with the behavior of hours in the U.S. data. Thus, these results suggest that the one-sector growth model can exhibit indeterminacy and generate second moment that are consistent with those observed in the U.S. data for plausible parameter values.

3. Conclusions

This note studies how the determinacy of competitive equilibrium depends on the magnitude of income effect on the demand for leisure. The note quantitatively characterizes regions of uniqueness and regions of indeterminacy of equilibria as functions of the magnitude of the income effect. This demonstrates the dependence of equilibrium indeterminacy on the degree of income effect. The analysis is carried out using the Jaimovich and Rebelo [15] utility function, which spans the entire range of income effect that exists between the two most widely used utility functions in the business cycle literature: the King, Plosser, and Rebelo [19] and the Greenwood, Hercowitz, and Huffman [12] utility functions.

The note suggests that, when the model is calibrated with levels of aggregate returns to scale that are within recent empirical estimates, there is a wide range of degrees of income effect that lie between the KPR and GHH values, giving rise to equilibrium indeterminacy. For these regions of indeterminacy, I simulate the model driven solely by sunspot shocks. I find that the second-moment properties of this model are generally consistent with the U.S. data at the business cycle frequency. These results suggest that the one-sector growth model can exhibit indeterminacy and generate second moment that are consistent with those observed in the U.S. data for plausible parameter values.

[22] Wen [30] provides an excellent discussion on how the low volatility of consumption relative to output in his model (as well as in that of Farmer and Guo [11]) leads to a very volatile investment path. Because the BFJR model belongs to the class of one-sector growth models that he considers, his discussion applies here as well.

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